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Abstract :

This deliverable presents two control schemes specifically developed within the coalitional framework foreseen as a part of the DYMASOS project, to address the establishment of coalitions among the diverse agents involved in systems-of-systems. Global optimality concerns are tackled through a top-down approach, whereas individual interests are addressed with a novel bottom-up coalition formation process, based on the allocation provided by the Shapley value.
A graph-based interpretation of the autonomous coalition formation process is given as a result of WP2 *Economics-driven coordination and market-based management of systems of systems*.
Furthermore, a population-based setting, considered within Task 1.3 of WP1 *Population dynamics based approach to the management of systems of systems*, is presented for the study of mechanisms for the convergence to Nash equilibria of local cooperating agents.
The stability issues — on both classic and coalition-wise sense — of SoS controlled by the coalitional schemes developed over Task 3.2 of WP3 are addressed as well in this document. Moreover, as part of Task 3.3, a discussion of the role of information exchange between the different agents acting within a SoS is included.

Keywords :

Coalitional control, game theory, distributed control, hierarchical control, system partitioning, systems-of-systems

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The DYMASOS Project

The well-being of the citizens in Europe depends on the reliable and efficient functioning of large interconnected systems, such as electric power systems, air traffic control, railway systems, large industrial production plants, etc. Such large systems consist of many interacting components. The sub-systems are usually managed locally and independently, according to different policies and priorities. The dynamic interaction of the locally managed components gives rise to complex behaviour and can lead to large-scale disruptions as e.g. black-outs in the electric grid.

Large interconnected systems with autonomously acting sub-units are called systems of systems. DYMASOS addresses systems of systems where the elements of the overall system are coupled by flows of physical quantities, e.g. electric power, steam or hot water, etc.

Within the project, new methods for the distributed management of large physically connected systems with local management and global coordination will be developed.

The DYMASOS Consortium consists of:

Participant no.	Participant organisation name	Participant short name	Country
1	Technische Universität Dortmund	TUDO	Germany
2	BASF SE	BASF	Germany
3	HEP-Operator distribucijskog sustava d.o.o	HEP	Croatia
4	INEOS Köln GmbH	INEOS	Germany
5	University of Seville	USE	Spain
6	University of Zagreb - Faculty of Electrical Engineering and Computing	UNIZG-FER	Croatia
7	ETH Zürich	ETH	Switzerland
8	RWTH Aachen University	RWTH	Germany
9	inno TSD	inno	France
10	Optimizacion Orientada a la Sostenibilidad SL	IDENER	Spain
11	euTeXoo GmbH	TEX	Germany
12	Ayesa Advanced Technologies SA	Ayesa AT	Spain

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1 Executive summary

This deliverable resumes the work carried out along the guidelines of Tasks 3.2, 3.3 (for which preliminary results were presented in deliverables D3.2 and D3.3) and 3.4 of WP3 *Coalitional Games in Systems of Systems*, concerning the development of methods for coalition formation among the agents of a system of systems (SoS).

An aspect so far rarely contemplated in distributed control problems is the explicit consideration of individual (local) interests of the components of a complex system. Indeed, in order to allow fundamental properties of centralized control, such as system-wide optimality and stability, the majority of the literature about distributed control focuses on the overall system performance. However, when dealing with systems with a strong heterogeneous character, selfish interests may not be neglected. One of the critical points that have to be kept into consideration is the *diversity* characterizing the systems in object, yielding very complex interactions between the agents involved (see, e.g., the AYESA and HEP case studies contemplated within the DYMASOS project). In order to tackle such inherent aspects of SoS, two basic architectures are contemplated within the WP3 task of DYMASOS project: *(i) top-down*, i.e., hierarchically supervised coalitional structure evolution, and *(ii) bottom-up*, i.e., autonomous negotiation between agents, leading to the emergence of cooperating clusters.

The present document is organized as follows: preliminaries are provided in Section 3 and 4, followed by the presentation of the two aforementioned classes of coalitional control architectures (Sections 5 and 6), developed within WP3 by the USE team. A coalition matching mechanism for coalition formation (developed by the FER Zagreb team from WP2) is presented at the end of Section 6. Section 7 presents an algorithm for the minimization of the disturbance resulting from subsystems' coupling (USE). Section 8 presents the population-based coalitional scenario developed within Task 1.3 of WP1 by the ETH Zurich team. Section 9 discusses the formation of coalitions of controllers from the information requirements point of view (USE). Finally, Sections 10 and 11 concern the stability issues encountered in the coalitional control framework, respectively on the coalition-wise and classical control theoretic aspects (USE).

2 Introduction

Logistical issues and structural constraints significantly define the character of systems of systems (SoS). A central issue when designing specific control strategies for SoS is that some of its components may have selfish interests, or may be concerned with privacy—such as to hinder the exchange of local information across the whole system. This issue becomes manifest when coupling between subsystems is not negligible, and non-local information is critical for adequate control feedback [1–3].

In spite of the huge effort dedicated to the development of distributed controllers for large-scale systems (see, e.g., [4–7]), the analysis of the relevance (and possibly of the cost) of the information exchange required by distributed control algorithms has received so far little attention. In order to allow funda-

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mental properties of centralized control, such as system-wide optimality and stability, the majority of the literature about distributed control rather overlooked privacy-related issues in order to focus on the overall system performance. However, when dealing with systems characterized by a strong heterogeneity, selfish interests may not be neglected.

The classification of controllers in the literature into either distributed or decentralized algorithms can be mainly related to the degree of dynamic interaction between the controlled subsystems: in the first class the agents need to communicate to coordinate their operations [6,8–10], whereas in the second class the limited degree of interaction allows the agents to tackle their control tasks with no need of communication [11,12]. Intuitively, the stronger the dynamic interaction among different parts of a system, the denser the communication required between the associated control agents. In several cases the variables of a system can be grouped to highlight weakly coupled blocks, often revealing a natural topology arising from the physical structure of the system [13–16]. In general, it is desirable to "localize" control laws following such topology—for ease of implementation and reduction of the communication overhead. From the coalitional control standpoint then it is arguably critical to characterize the improvement provided by a broader knowledge of the system, and promote the formation of coalitions accordingly. When only a subset of the control agents is willing to exchange information about their subsystems, an interesting point to investigate is the performance bound of the control loop possibly achievable with *partial system information*. On such line of research, studies have been carried out by [15,17].

Another interesting challenge is the online identification of subsystems' coupling degree [18], and the consequent adjustment of the control topology (thus varying the associated computational and communicational requirements). In coalitional control, the formation of coalitions is based upon the varying conditions of coupling between the subsystems [19].

In such settings, information about the relative relevance of the agents can be employed to efficiently direct the system's resources where most needed [20,21]. Cooperative game theory currently represents a basic tool to derive ad-hoc metrics [22,23].

Thanks to modern network technologies, such as wireless networks, smart sensors, and database possibilities offered by cloud computing, a huge amount of diverse information can be shared across a large-scale system in an efficient and flexible way [24]; also, following the wide diffusion of computationally capable mobile devices, the possibilities offered by human-in-the-loop control systems have been enhanced [25]. All these factors contribute to impulse a new approach to distributed control problems, where the cooperation between networked controllers is actively fostered and adapted in real-time to the state of the system.

The advances introduced by these new technologies can be already identified on infrastructure systems, e.g., in modern traffic, water, and electricity networks [5,26], emphasizing how central is the role that information plays in their efficient management. However, in the majority of the studies regarding the control of networked systems the focus has been kept on issues related with the transmission medium itself, e.g., limited bandwidth, data loss, noisy channel [27].

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Besides the smart grid (see [1] and references therein), a clear example in this direction is the great interest towards the utilization of Intelligent Transportation Systems (ITS). A consistent research effort is being devoted to this topic, typically involving different kind of game models in order to grasp the complex phenomena derived by the interaction of its heterogeneous user population. Some examples are the analysis of the problem of choosing the EV charging station [26], the study of the consequences of a coalitional scenario among charging managers [28], or the setting for enabling Vehicle-to-Grid (V2G) operations through coalitions of users [29].

In this document, two different approaches—hierarchically supervised and autonomous—to *coalitional control* are presented. More specifically, according to such control schemes, the structure of each agent's model predictive controller [30, 31] is adjusted following the time-variant coupling conditions. The first, discussed in Section 5, is a *top-down* approach, where the global coalitional structure is optimized at a supervisory layer. To address individual rationality instead, a *bottom-up* approach is proposed in Section 6: here the formation of coalitions is produced as the outcome of an autonomous bargaining procedure, following ad-hoc criteria—formulated on the basis of both cooperative control and game-theoretical fundamentals [18, 32]. Further aspects of coalition formation in control, such as information-related issues, or stability—both system-wise and coalition-wise—are addressed in the last part of the document. Next, a description of the considered setting is provided.

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3 Preliminaries

3.1 System description

Let us consider a SoS whose components are identified within the set $\mathcal{N}$. Each subsystem $i \in \mathcal{N}$ is governed by a local controller, and its behavior follows the linear discrete-time model

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + D_i w_i(k), \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{q_i}$ are respectively the local state and input vectors, constrained in the sets $\mathcal{X}_i$ and $\mathcal{U}_i$ respectively. The vector $w_i \in \mathbb{R}^{m_i}$ represents the measurable disturbances resulting from the coupling with other subsystems,

$$w_i(k) = \sum_{j\in\mathcal{M}_i} A_{ij}x_j(k) + B_{ij}u_j(k) \triangleq \sum_{j\in\mathcal{M}_i} d_{ij}(k), \tag{2}$$

where $\mathcal{M}_i \triangleq \{j \in \mathcal{N} \setminus \{i\} | A_{ij} \neq \mathbf{0} \vee B_{ij} \neq \mathbf{0}\}$ is referred to as the *neighborhood* of subsystem i.

Denoting the global state as $x \equiv \{x_i\}_{i\in\mathcal{N}} \in \mathbb{R}^n$ and the global input as $u \equiv \{u_i\}_{i\in\mathcal{N}} \in \mathbb{R}^q$, the whole SoS is modeled by

$$x(k+1) = Ax(k) + Bu(k), \tag{3}$$

where $A = \{A_{ij}\}_{i,j\in\mathcal{N}} \in \mathbb{R}^{n\times n}$ and $B = \{B_{ij}\}_{i,j\in\mathcal{N}} \in \mathbb{R}^{n\times q}$.

Analogous models have been employed for the control of real large-scale systems such as drinking water networks composed by interconnected water tanks [33], irrigation canals, modeled by integrator-delay models in [19, 34, 35], supply chains [36], traffic networks and power grids [37].

3.2 Exchange of information

The set of controllers can communicate through a network infrastructure that can be schematized by the graph $\mathcal{G}(k) = (\mathcal{N}, \mathcal{E}(k))$, where $\mathcal{E}(k) \subseteq \mathcal{N} \times \mathcal{N}$ is the set of links. The time dependency of $\mathcal{E}(k)$ reflects the possibility to activate or shut down data links at any given time step k. The description provided by $\mathcal{G}(k)$ delineates a partition $\mathcal{P}(\mathcal{N}, \mathcal{G}(k)) = \{\mathcal{C}_1, \dots, \mathcal{C}_{N_c}\}$ of the the set of controllers into N_c connected components, referred to as coalitions [38]. Coalitions are disjoint sets such that [39]

$$\mathcal{C}_i \subseteq \mathcal{N},\ \forall i \in \{1, \dots, N_c\}, \text{ and } \bigcup_{i=1}^{N_c} \mathcal{C}_i = \mathcal{N}.$$

The number of coalitions N_c pertains to the interval $[1, |\mathcal{N}|]$, whose extremes correspond to the centralized control case (all the $|\mathcal{N}|$ subsystems connected[1]) and the case where each subsystem "forms a coalition" on its own (all links disabled).[2] For the sake of readability, let us define the set $\mathcal{S}_{\mathcal{P}}(\mathcal{N}, \mathcal{G}(k)) = \{1, \dots, N_c\}$

[1]Notice that this condition does not necessarily require all the links to be active.
[2]The $|\cdot|$ operator stands for the cardinality of a set.

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indexing the coalitions characterizing the current partition of the system. The dynamics (1) of all subsystems relative to a given connected component $i \in \mathcal{S}_{\mathcal{P}}$ can be aggregated as

$$\xi_i(k+1) = \mathbb{A}_{ii}\xi_i(k) + \mathbb{B}_{ii}\nu_i(k) + \mathbb{D}_i\omega_i(k), \tag{4}$$

with $\xi_i = \{x_j\}_{j\in\mathcal{C}_i}$ the aggregate state vector, and $\mathbb{A}_{ii} = [A_{jl}]_{j,l\in\mathcal{C}_i}$ the relative state transition matrix, describing the state coupling between members of the same coalition. The vector ν_i and the matrix $\mathbb{B}_{ii}$ are derived by an analogous definition. Finally, the vector

$$\omega_i = \{w_j\}_{j\in\mathcal{C}_i} \tag{5a}$$

gathers the disturbances due to the coupling with subsystems external to $\mathcal{C}_i$. Following (2) it holds that

$$w_j = \sum_r d_{jr}(k), \text{with } r \in \mathcal{M}_j \setminus (\mathcal{C}_i \cap \mathcal{M}_j), \tag{5b}$$

pointing out how, for each $j \in \mathcal{C}_i$, the set of unknown coupling from neighboring subsystems is reduced to the neighbors left out of the coalition. That is, from the coalition standpoint, the uncertainty comes from any subsystems $r \in (\bigcup_{j\in\mathcal{C}_i} \mathcal{M}_j) \setminus \mathcal{C}_i$. The composition of $\mathbb{D}_i$ follows accordingly.[3]

3.3 Control objective

We assume in the remainder that the objective of the controller is to drive the system's state toward the origin of the state space. The cost of subsystem i at any given time step k, as a function of the distance to its setpoint and the control effort, is expressed by

$$\ell_i(k) = x_i(k)^\intercal Q_i x_i(k) + u_i(k)^\intercal R_i u_i(k), \tag{6}$$

where $Q_i \in \mathbb{R}^{n_i\times n_i}$ and $R_i \in \mathbb{R}^{q_i\times q_i}$ are positive (semi-)definite weighting matrices. The stage cost (6) is extended to any $\mathcal{C}_i \subseteq \mathcal{N}$ as

$$\boldsymbol{\ell}_i(k) = \xi_i(k)^\intercal \mathbf{Q}_i \xi_i(k) + \nu_i(k)^\intercal \mathbf{R}_i \nu_i(k); \tag{7}$$

notice that if $\mathbf{Q}_i = \operatorname{diag}\{Q_j\}_{j\in\mathcal{C}_i}$, and $\mathbf{R}_i = \operatorname{diag}\{R_j\}_{j\in\mathcal{C}_i}$, then $\boldsymbol{\ell}_i \equiv \sum_{j\in\mathcal{C}_i} \ell_j$.

Local controllers aggregate into a coalition with the aim of coordinating the effort and achieving a better overall performance (7). At time k, a control sequence for all subsystems $j \in \mathcal{C}_i$ is derived by the

[3] In case of singleton coalition, i.e., $\mathcal{C} \equiv \{i\}$, the description given by (4) coincides with (1).

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joint solution of the model predictive control (MPC) problem

$$\min_{\boldsymbol{\nu}_i} \mathbf{J}_i = \sum_{t=0}^{N_p-1} \boldsymbol{\ell}_i(t|k) + \xi_i(N_p|k)^\intercal \mathbf{P}_i \xi_i(N_p|k) \tag{8a}$$

s.t.

$$\xi_i(t+1|k) = \mathbb{A}_{ii}\xi_i(t|k) + \mathbb{B}_{ii}\nu_i(t|k) + \mathbb{D}_i\omega_i(t|k), \tag{8b}$$

$$\xi_i(t|k) \in \Xi_i,\ t = 0, \ldots, N_p, \tag{8c}$$

$$\nu_i(t|k) \in \Psi_i,\ t = 0, \ldots, N_p - 1, \tag{8d}$$

$$\xi_i(0|k) \equiv \xi_i(k), \tag{8e}$$

$$\omega_i(t|k) = \hat{\omega}_i(k+t),\ t = 0, \ldots, N_p - 1, \tag{8f}$$

where $\Xi_i = \mathcal{X}_1 \times \mathcal{X}_2 \times \ldots \mathcal{X}_{N_i}$ is the Cartesian product of the local state constraints relative to each member of the coalition (an analogous definition holds for the input constraint set Ψ_i). The symmetric positive definite matrix $\mathbf{P}_i$ is the terminal weight in (8). The minimizer of $\mathbf{J}_i$ over the prediction horizon N_p in (8a) is the column vector

$$\boldsymbol{\nu}_i(k)^* = [\nu_i(0|k)^*, \nu_i(1|k)^*, \ldots, \nu_i(N_p-1|k)^*] \,.$$

At time k, the first element of $\boldsymbol{\nu}_i(k)^*$, namely $\nu_i(0|k)^* \equiv \{u_j(0|k)^*\}_{j\in\mathcal{C}_i}$, is applied to every subsystem involved in the coalition. The control inputs for successive time steps are obtained by solving (8) in a receding horizon fashion. Notice that, in absence of measures from subsystems external to the coalition—since problem (8) is solved independently for each coalition $\mathcal{C}_i \in \mathcal{P}(\mathcal{N}, \mathcal{G}(k))$—an estimate of the disturbance term (5) is employed, as stated in (8f).

3.4 Cost of cooperation

Cooperation may not come for free. In general, it is rather expected that the effort required for the coordination is proportional to the number of agents involved in a coalition, requiring as well non-negligible data exchange. Therefore, costs required for the cooperation of a given set of agents can be taken into account by means of ad-hoc indices related with, e.g., the size of the coalition, the number of data links needed in order to establish communication between every member of the coalition.[4] Further measures may be employed to evaluate cooperation costs, based on, e.g., the number of decision variables and/or constraints of the aggregate problem, reflecting the computational requirements.

We assume such cooperation costs comparable with the stage cost (7). For coalition $\mathcal{C}_i$,

$$\chi_i(k) = f(|\mathcal{C}_i|, |\mathcal{E}_i(k)|), \tag{9}$$

where $\mathcal{E}_i(k) \subseteq \mathcal{E}(k)$ is the subset of edges of the graph $\mathcal{G}(k)$ connecting the nodes in $\mathcal{C}_i$. We can thus rectify what stated in Section 3.3, as "local controllers aggregate into a coalition with the aim of coordinating

[4]For the definition of connected component the reader is referred to Section 3.2.

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the effort and *achieving the best tradeoff* between the performance (7) and the associated cooperation costs (9)".

3.5 Global control problem

The overall control problem can be stated as

$$\min_{\nu,\mathcal{E}} \sum_{i\in\mathcal{S}_{\mathcal{P}}} \mathbf{J}_i(\xi_i(k),\nu_i) + \mathbf{J}_i^{\chi}(\mathcal{E}) \tag{10a}$$

s.t.

$$\xi_i(t+1|k) = \mathbb{A}_{ii}\xi_i(t|k) + \mathbb{B}_{ii}\nu_i(t|k) + \mathbb{D}_i\omega_i(t|k), \tag{10b}$$

$$\xi_i(t|k) \in \Xi_i,\ t = 0,\ldots,N_p, \tag{10c}$$

$$\nu_i(t|k) \in \Psi_i,\ t = 0,\ldots,N_p - 1, \tag{10d}$$

$$\xi_i(0|k) \equiv \xi_i(k), \tag{10e}$$

$$\omega_i(t|k) = \hat{\omega}_i(k+t),\ t = 0,\ldots,N_p - 1, \tag{10f}$$

$$\mathcal{E}(t) \subseteq \mathcal{N}\times\mathcal{N},\ t = 0,\ldots,N_p, \tag{10g}$$

$$\mathcal{E}(t) = \mathcal{E}(0),\ t = 1,\ldots,N_p, \tag{10h}$$

where, for any $i \in \mathcal{S}_{\mathcal{P}}$,

$$\mathbf{J}_i^{\chi}(\mathcal{E}) = \sum_{t=0}^{N_p} \chi_i(t). \tag{11}$$

Notice that, according to constraints (10g) and (10h), we assume the set of edges $\mathcal{E}$—hence the system partition $\mathcal{P}(\mathcal{N},\mathcal{G})$—constant during the prediction horizon $t \in [k, k+N_p]$. Problem (10) constitutes a dynamic optimization with mixed integer variables, which is generally not practical to solve. Since any given $\mathcal{E}$ corresponds to a partition of the global system, the composition of the resulting coalitions' state and input vectors and matrices will implicitly depend on it. The choice of the network topology is made within a discrete set whose size grows exponentially with the number of subsystems (implying a higher number of links). The control architectures presented in the remainder can provide a suboptimal, yet less computationally expensive solution.

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4 Coalition formation

In the considered setting, the control agents can decide with whom to cooperate and under which conditions (namely, the allocation of the payoffs derived from the cooperation). We model such situation as a coalitional game. A coalitional game is uniquely defined by the pair $(\mathcal{N}, v)$, where $\mathcal{N}$ is the set of players and v is the *value* of a given coalition.

Coalition formation involves three main steps. The first two are *(i)* generation of the coalition structure and *(ii)* solution of the optimization problem for each coalition [29,40]. Coalition formation is commonly studied in the form of *characteristic function* games, where a value is assigned to any possible coalition $\mathcal{C} \subseteq \mathcal{N}$ through a function $v : 2^{\mathcal{N}} \rightarrow \mathbb{R}$. Given the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ describing the associations among the control nodes of the system, the value of a coalition structure $\mathcal{P}(\mathcal{N}, \mathcal{G}) \equiv \{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{N_c}\}$ is defined as its aggregate value, i.e.,

$$V(\mathcal{P}) = \sum_{i \in \mathcal{S}_{\mathcal{P}}} v(\mathcal{C}_i), \tag{12}$$

where $\mathcal{S}_{\mathcal{P}} = \{1, \dots, N_c\}$. The optimal coalition structure $\mathcal{P}^*$ is found as the one characterized by the highest value V^*. However, such problem has been demonstrated to be NP-complete [40]. To overcome this issue, several solutions—resorting to heuristics, dynamic-programming, branch-and-bound algorithms, etc.—have been proposed in the literature (see [22, 29] and references therein). One such solution—a hierarchical scheme that manipulates the controller topology with regard to both the current state of the system and the communication cost—is presented in Section 5. According to this scheme, the overall system is partitioned into coalitions working in a decentralized fashion, within which the agents are able to communicate with those other agents whose cooperation is most relevant.

Particularly interesting when local interests are of main importance, the third and final step consists in the *(iii)* distribution of the value of a coalition among its members. The *payoff* ϕ_i is defined as the utility received by each agent $i \in \mathcal{C}$ by the division of $v(\mathcal{C})$. The vector of payoffs assigned to all the agents is referred to as the *allocation*. Of course, this third step is only possible if the real value $v(\mathcal{C})$ associated with coalition $\mathcal{C} \subseteq \mathcal{N}$ can be divided and transferred among its members (e.g., side-payments used to attract other players).

Remark 1 *In characteristic form games, the value of a given coalition depends only on the members that compose it, with no regard to how the rest of the agents are organized. Such model does not apply to the vast majority of real life applications. Indeed, although games in characteristic form provide a means of modeling a wide spectrum of scenarios, it is natural in engineering applications to encounter problems in which the value of a given coalition cannot be determined regardless of how the rest of the agents are organized. Games in* partition form *can model this type of problems [41].*

In games in *partition form*, given a partition $\mathcal{P} = \{\mathcal{C}_1, \dots, \mathcal{C}_l\}$ of $\mathcal{N}$, the value of any coalition $\mathcal{C}_i \in \mathcal{P}$ is expressed as $v(\mathcal{C}_i, \mathcal{P})$. However, it is not generally possible to derive a closed-form allocation in the considered scenario. Hence, we approximate the partition function game with a characteristic function game

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by assigning values to coalitions following an heuristic approach. For example, if a minmax approach is employed, the value of a given coalition will take into account the most unfavorable externalities given by any coalitional setup of the rest of agents.

The redistribution of the coalitional value is addressed here through the autonomous bargaining approach presented in Section 6, and the utility transfer algorithm introduced in Section 10.

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5 Supervised coalition structure generation

Often different parts of a networked system are owned and managed by independent entities (think about infrastructures), unwilling to coordinate their action unless strictly necessary. In addition, permanent communication across the entire system network can be impractical. Consequently, even when the whole system is owned and managed by a single entity, the use of a traditional centralized control approach is hampered. This motivates the two-layer hierarchical control scheme presented in this section. The basic setting is described in Section 3. The algorithm presented in the remainder is an approximation of Problem (10a) detailed in Section 3.5, for the generation of the optimal coalitional structure according to (12).

The main goal of the supervisory layer is to find the best compromise between control performance and communication costs by actively modifying the network topology. The actions taken at the supervisory layer alter the control agents' knowledge of the complete system, and the set of agents with which each one of them can communicate. The data links that do not yield a significant improvement of the control performance, compared with their relative cost of use, are disconnected. This feature is particularly interesting, e.g., for communication infrastructures based on battery-powered wireless devices. Each group of linked subsystems—a *coalition*—is independently controlled following a decentralized MPC scheme, constituting the bottom layer of the architecture [19].

The basic idea is to partition the centralized problem over a given number of local controllers or *agents* (see, e.g., [13]). A globally-designed feedback control law associated to such partition will imply the satisfaction of certain global properties. Given such feedback law, the cooperation of the agents may be situated at two different levels: it can turn into an explicit exchange of local information (such as interaction models, state measures, etc.) for the joint optimization of future input sequences, or it can remain at the implicit level dictated by the globally-designed control law.

A multi-agent control scheme based on the same basic idea is discussed in [22], where a design method with closed-loop stability guarantees is provided. The network topology optimization problem is posed as a cooperative game, in order to study the relevance of the different links and agents (in view, e.g., of fault-tolerant policies) under a game-theoretical perspective. From this perspective, the sequence of optimal network topologies is interpreted as a coalition of links that evolves in order to optimize the expected evolution of the closed-loop system.

Further related works are [42], addressing the formation of groups of cooperative agents according to the coupling constraints that are active at a given time; the work of [43] describes a hierarchical framework where information among the agents is exchanged at each time step within clusters of strongly dynamically coupled subsystems, while a slower communication rate is required between different clusters. In [33], the complexity of the model predictive control problem of the Barcelona drinking water network is reduced by means of a partitioning algorithm, and then control the resulting subnetworks in a hierarchical-distributed manner. In [44] a flexible hierarchical MPC scheme is proposed for a hydro-power valley, where the priority of the agents in optimizing their control actions can be rearranged according to the different operational conditions.

The scheme presented here focuses on how the dynamic perturbation from one subsystem to another varies with time [19]. A cost on the coordination effort is considered as a function of the data link usage, so that the overall structure of the controller evolves by trading off performance for savings on communication costs.[5] As a result, coordination between agents is promoted whenever the dynamic interaction between their corresponding subsystems is over the threshold dictated by the allowed communication costs.

5.1 System description

The dynamics of any subsystem $i \in \mathcal{N}$ are described by the linear model (reported from Section 3.1):

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k), \tag{13a}$$

$$w_i(k) = \sum_{j \in \mathcal{M}_i} A_{ij}x_j(k) + B_{ij}u_j(k), \tag{13b}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{q_i}$ are the state and input vectors respectively, and $w_i \in \mathbb{R}^{m_i}$ describes the influence on x_i of the neighbors' states and inputs. In (13b), $x_j \in \mathbb{R}^{n_j}$ and $u_j \in \mathbb{R}^{q_j}$ are the state and input vectors of each neighbor $j \in \mathcal{M}_i$ of subsystem i. The neighborhood set $\mathcal{M}_i$ is defined as:

$$\mathcal{M}_i = \{j \in \mathcal{S} | A_{ij} \neq \mathbf{0} \vee B_{ij} \neq \mathbf{0}, j \neq i\}, \tag{14}$$

i.e., it contains any subsystem $j \neq i$ whose state and/or input produce some effect on the dynamics of subsystem i.

5.2 Exchange of information

All the control agents can communicate through a data network whose topology is described by means of the undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where to each subsystem in $\mathcal{N}$ is assigned a node.[6] Each link $\ell_{ij} = \{i,j\} = \{j,i\} = \ell_{ji} \in \mathcal{N} \times \mathcal{N}$ can be either enabled or disabled. Then the *network topology* $\mathcal{E}(k) \subseteq \mathcal{N} \times \mathcal{N}$ is defined as the set of links enabled at a given time, i.e., $\ell_{ij} \in \Lambda(k)$ if and only if it is used at time k. Each active link has a cost $c_\ell > 0$ per time of use. This cost can vary, e.g., as a function of the available bandwidth.

As agents within the same communication component will benefit from cooperation—sharing information in order to aggregate their control tasks—we will refer to such components as coalitions, and the partition resulting by a given network topology $\mathcal{E}(k)$ will be denoted as $\mathcal{P}(\mathcal{N}, \mathcal{G}(k)) = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{N_c}\}$. The set of indices $\mathcal{S}_\mathcal{P} = \{1, \ldots, N_c\}$ is defined as well.

[5] Further criteria may be employed to evaluate cooperation costs, based on, e.g., the total number of agents involved in the coalition, the number of decision variables and/or constraints of the aggregate problem, reflecting the computational requirements [45].

[6] For further details, see Section 3.2.

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5.3 Coalition dynamics

In order to describe the dynamics of each coalition $\mathcal{C}_i \in \mathcal{P}$, the following extension of (13a) holds:[7]

$$\xi_i(k+1) = \mathbb{A}_{ii}\xi_i(k) + \mathbb{B}_{ii}\nu_i(k) + \mathbb{D}_i\omega_i(k), \tag{15}$$

where ξ_i and ν_i are respectively the state and input vectors of coalition $\mathcal{C}_i$, composed by stacking the vectors of all the subsystems in the coalition. As an extension of (13b), ω_i expresses the influence of *neighboring* coalitions on ξ_i, as defined in (5a).

5.4 Control objective

In the application considered here, the control objective is to regulate the state of all subsystems while minimizing a cost that depends on the state and input trajectories. An additional term in the cost function will take into account the use of network resources. Thus, the cost function is divided into a term $\mathbf{J}_i$, $i \in \mathcal{S}_\mathcal{P}$, representing the optimal performance objective and a term $J_{n,j}$, $j \in \mathcal{N}$, expressing the network-related cost:

$$\mathbf{J}_i = \sum_{t=0}^{N_p-1} (\xi_i^\intercal(t|k)\mathbf{Q}_i\xi_i(t|k) + \nu_i^\intercal(t|k)\mathbf{R}_i\nu_i(t|k)) + \xi_i^\intercal(N_p|k)\mathbf{P}_i\xi_i(N_p|k), \forall i \in \mathcal{S}_\mathcal{P}, \tag{16a}$$

$$J_{n,j} = N_p\frac{c_\ell}{2}n_{\ell,j}\left(\mathcal{E}(k)\right), \forall j \in \mathcal{N} \tag{16b}$$

where the notation $x(t|k)$ corresponds to the value of x predicted at time $k+t$, based on the knowledge at time k, and $\mathbf{Q}_i \geq 0$, $\mathbf{R}_i > 0$ and $\mathbf{P}_i = \mathbf{P}_i^\intercal > 0$ are constant weighting matrices. In (16b), $n_{\ell,j}\left(\mathcal{E}(k)\right)$ is the number of active links *directly* connecting agent j to other agents according to $\mathcal{E}(k)$; the network topology $\mathcal{E}(k)$ is kept constant during the prediction horizon. Note that each agent shares the cost of a link with the agent located at the other side of that link. Then the overall optimization problem can be posed in the form of (10), described in Section 3.5. In particular, the objective function can be stated as:

$$\min_{\boldsymbol{\nu},\mathcal{E}} \sum_{i\in\mathcal{S}_\mathcal{P}(\mathcal{E})} \mathbf{J}_i(\xi_i(k),\nu_i) + \sum_{j\in\mathcal{N}} J_{n,j}\left(\mathcal{E}\right), \tag{17}$$

which will be subject to constraints (10b)–(10h).

As already mentioned in Section 3.5, Problem (17) constitutes a dynamic optimization problem with mixed integer variables. Since any topology corresponds to a partition of the global system, the composition of the resulting coalitions' state and input vectors and matrices will implicitly depend on $\mathcal{E}$. The choice of the network topology is made within a discrete set whose size grows exponentially with the number of subsystems. Next, we present a hierarchical multi-agent control algorithm that provides a suboptimal, yet less computationally expensive solution.

[7]See also Section 3.2.

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5.5 The control algorithm

The architecture of the proposed approximation of (17) is organized on two layers: the top layer takes charge of the choice of the network mode, whereas the bottom layer handles the estimation and the real-time control tasks. At the bottom layer, the control is decentralized into the coalitions arising from the optimal (according to (17)) partition $\mathcal{S}^*_{\mathcal{P}}$. With the term *decentralized* we designate the complete absence of communication among different coalitions; agents within a coalition share their information at each sample time. As a consequence, the term ω_i modeling the effect of neighboring coalitions cannot be computed through (5a), and each coalition needs to employ an estimate $\hat{\omega}_i$. Issues related with such estimation are strongly case-related, and outside the scope of this document. In general, it is desirable that inter-coalition coupling show relatively slow dynamics, such that transient phenomena can be neglected. Next, further details are given about the operation of the top layer.

The discrete part of (17), constituting the most computationally demanding part of the problem, is handled at the top layer. For this reason, its solution is computed on a coarser time scale (w.r.t. the sample time required for the control of the system), and the resulting topology maintained during the following interval $T_{\mathcal{E}}$. In order to select the most appropriate global control structure, several network topologies are compared. Let $\mathfrak{E}^+ = \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_{N_{\mathcal{E}}}\}$ be the set of possible network topologies to be evaluated. Then, let us define the function $J : \mathbb{R}^n \times (\mathcal{N} \times \mathcal{N}) \mapsto \mathbb{R}$ as follows [46]:

$$J(\xi, \mathcal{E}) = \sum_{i \in \mathcal{S}_{\mathcal{P}}(\mathcal{E})} \xi_i^{\mathsf{T}} \mathbf{P}_i \xi_i + c_\ell |\mathcal{E}| T_{\mathcal{E}}, \; \mathcal{E} \in \mathfrak{E}^+, \tag{18}$$

where $P_i = P_i^{\mathsf{T}} > 0$, $|\mathcal{E}|$ is the number of enabled links and c_ℓ is the cost of use of one link, considered over the interval $T_{\mathcal{E}}$. In the remainder, the vector $\xi \in \mathbb{R}^n$, with $n = \sum_{i \in \mathcal{S}_{\mathcal{P}}} n_i$, will designate the global vector obtained by stacking the state vectors ξ_i of all coalitions $\mathcal{C}_i$, $\forall i \in \mathcal{S}_{\mathcal{P}}$. Notice that ξ coincide with a rearranged global state vector $x \in \mathbb{R}^n$.

It is not pragmatic to see $\mathfrak{E}^+$ as the set containing every possible configuration of links. Because the number of all possible topologies grows exponentially with the number of subsystems, the set $\mathfrak{E}^+$ should be defined as a reasonably sized set of *relevant* topologies for the system to be controlled. The composition of $\mathfrak{E}^+$ could either be static or evolving in relation with, e.g., the current state of the system, the network constraints, or the willingness to cooperate among the agents. Of all the configurations considered at a given moment, the one giving the optimal value of (18), denoted as $\mathcal{E}^* \in \mathfrak{E}^+$, will be applied during the next interval $T_{\mathcal{E}}$.

As a consequence to the choice of any given topology $\mathcal{E}$, the set of agents is partitioned into a specific set of coalitions $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_{N_c}\}$. To attain the optimal performance objective (16a), a feedback gain $\mathbb{K}$ for the whole system is computed at the top layer.[8] In conformity with the system partition $\mathcal{P}(\mathcal{N}, \mathcal{G}(k))$, $\mathbb{K}$ will be composed of a set of decentralized feedback gains, each one associated to a coalition, i.e., $\mathbb{K} = diag\{K_1, \ldots, K_{N_c}\}$. Let $\mathbb{P} > 0$ be the block matrix having $\{P_1, \ldots, P_{N_c}\}$ on its diagonal, and

[8]Note that, for the grand coalition, the feedback law $\mathbb{K}$ will coincide with the LQR gain.

consider the Lyapunov function $V(\xi) = \xi^{\intercal}\mathbb{P}\xi$, where $\xi \triangleq \{\xi_i\}_{i\in\mathcal{S}_{\mathcal{P}}}$ is the global state vector, rearranged according to the partition $\mathcal{P}$. For $V(\xi)$ to constitute an upper bound on the infinite-horizon performance objective, the constraints of the following LMI problem have to be satisfied (see, e.g., [47]):

$$\max_{\mathbb{K},\mathbb{P}} \mathbf{Tr}\,\mathbb{P}^{-1} \tag{19}$$

s.t.

$$\mathbb{P} = \mathbb{P}^{\intercal} > 0,$$
$$(A_\xi + B_\nu\mathbb{K})^{\intercal}\mathbb{P}(A_\xi + B_\nu\mathbb{K}) - \mathbb{P} \leq -\mathbf{Q} - \mathbb{K}^{\intercal}\mathbf{R}\mathbb{K},$$

where A_ξ and B_ν are respectively the global state and input matrices, composed to match ξ and $\nu \triangleq \{\nu_i\}_{i\in\mathcal{S}_{\mathcal{P}}}$. Similarly, $\mathbf{Q} = diag\{\mathbf{Q}_1, \ldots, \mathbf{Q}_{N_c}\} \geq 0$ and $\mathbf{R} = diag\{\mathbf{R}_1, \ldots, \mathbf{R}_{N_c}\} > 0$ are the global weighting matrices.

By the solution of (19), a set of feedback control laws $\nu_i = K_i\xi_i$ that minimize $V(\xi)$ and a set of matrices P_i, is obtained for all $i \in \mathcal{S}_{\mathcal{P}}$. These matrices are then used to compute the value of (18) and find its minimizer $\mathcal{E}^*$.

Remark 2 *Notice that the evaluation of different network topologies is independent and can be executed in parallel on a multi-processor platform. Also, the set of control laws associated with any network topology could be stored and reused whenever the same topology is considered again, without the need of solving more than once the relative LMI problem.*

For further details the reader is referred to [19, 22].

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6 Autonomous coalition formation

In order to allow fundamental properties of centralized control, such as optimality and stability, the local interests of each component of a large-scale system are in practice subordinated for the overall system performance. On this same principle is based the hierarchical coalitional control architecture presented in Section 5, where a tradeoff between global optimality and inter-agent information exchange is followed. However, when dealing with systems characterized by a strong heterogeneous character, selfish interests may not be neglected. To address this issue, a bottom-up approach to coalitional control is proposed. Here the structure of each agent's model predictive controller (MPC) is allowed to evolve according to the time-variant coupling conditions of the system, by means of autonomous formation of coalitions, as outcome of a pairwise bargaining procedure.

6.1 System description

Consider a set $\mathcal{N} = \{1, \dots, M\}$ of discrete-time linear processes, such that each can be modeled by (1):

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + D_i w_i(k),$$

where the coupling between the subsystems' dynamics is modeled by the term w_i (see (2) in Section 3.1),

$$w_i(k) = \sum_{j \in \mathcal{M}_i} A_{ij}x_j(k) + B_{ij}u_j(k),$$

with the set $\mathcal{M}_i$ indexing all subsystem coupled to subsystem i.

6.2 Control objective

A central point in this discussion is the assumption that the agents—each of which is assigned the task of controlling one subsystem—act in order to minimize their local stage cost over a decentralized noncooperative MPC architecture. We recall the local stage cost (6) from Section 3.3:

$$\ell_i(k) = x_i(k)^\intercal Q_i x_i(k) + u_i(k)^\intercal R_i u_i(k), \tag{20}$$

where x_i and u_i are respectively the state and input vectors of subsystem i. The matrices Q_i and R_i weight the deviation of state and input from their reference.

From a game-theoretical perspective, the agents are assumed to be *rational* and *selfish*. The inputs

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$u_i(k)$ to the process are obtained, for each $i \in \mathcal{N}$, as the solution of the following optimization problem:

$$\min_{\mathbf{u}_i} \sum_{t=0}^{N_p-1} \ell_i(t|k) + x_i(N_p|k)^\intercal P_i x_i(N_p|k) \tag{21a}$$

s.t.

$$x_i(t+1|k) = A_{ii}x_i(t|k) + B_{ii}u_i(t|k) + D_i w_i(t|k), \tag{21b}$$

$$x_i(t|k) \in \mathcal{X}_i,\ t = 0,\dots,N_p, \tag{21c}$$

$$u_i(t|k) \in \mathcal{U}_i,\ t = 0,\dots,N_p-1, \tag{21d}$$

$$x_i(0|k) \equiv x_i(k), \tag{21e}$$

$$w_i(t|k) = \hat{w}_i(k+t),\ t = 0,\dots,N_p-1, \tag{21f}$$

The optimization variable

$$\mathbf{u}_i \triangleq [u_i(k),\dots,u_i(k+N_p-1)] \in \mathbb{R}^{(q_i N_p)\times 1}$$

is a column vector composed of the sequence of control actions along the prediction horizon of length N_p. At time k the first element $\mathbf{u}_i^*(0) \triangleq u_i(k)$ of the minimizing sequence is applied to the system, and the problem is solved again at subsequent time steps in a receding horizon fashion [30, 31].

Remark 3 *In case of absent/partial information exchange, the inputs and the state of neighboring agents are unknown and problem* (21) *has to be solved over an estimated $\hat{w}_i$. By means of the autonomous coalition generation framework considered in the remainder, agents will be able to expand their knowledge of the rest of the system and to jointly agree on the value assigned to the inputs.*

6.3 Autonomous coalition structure generation

In the remainder, the term *player* may refer to either a single control agent or a group of agents acting as a single entity, as a consequence of their participation in the same coalition. In order to keep the notation simple, indices $\{1, 2\}$ will be used to designate the players; furthermore, the notation $\{1\cup 2\}$ will refer to their merger. The subsystems involved in either part of a given bargaining process are identified by the sets $\mathcal{P}_1 \subset \mathcal{N}$ and $\mathcal{P}_2 \subset \mathcal{N}$: the coalitions of agents corresponding to these sets constitute the two players. The set of subsystems forming the merger will be designated as $\mathcal{P}_{1\cup 2} \triangleq \mathcal{P}_1 \cup \mathcal{P}_2$. Overlapping coalitions are not considered, so $\mathcal{P}_1 \cap \mathcal{P}_2 = \varnothing$.

The states and inputs of every subsystem taking part in the bargaining process are gathered into the player's state and input vectors, defined as:

$$\xi_1 \triangleq \{x_j\}_{j\in\mathcal{P}_1}, \qquad \upsilon_1 \triangleq \{u_j\}_{j\in\mathcal{P}_1},$$

$$\xi_2 \triangleq \{x_j\}_{j\in\mathcal{P}_2}, \qquad \upsilon_2 \triangleq \{u_j\}_{j\in\mathcal{P}_2},$$

where $\mathcal{P}_1 \subset \mathcal{N}$, $\mathcal{P}_2 \subset \mathcal{N}$, and $\mathcal{P}_1 \cap \mathcal{P}_2 = \varnothing$. Finally, the merger state and input vectors are composed according to

$$\xi_{12}^\intercal = [\xi_1^\intercal \ \xi_2^\intercal], \quad \upsilon_{12}^\intercal = [\upsilon_1^\intercal \ \upsilon_2^\intercal].$$

At this point, the objective is to establish a possible criterion for endogenous coalition formation, *oriented at networks of dynamically coupled systems*, providing redistribution of the benefits derived from the cooperation. We first point out the issues—from the individual control agent's standpoint—of the absence of redistribution of such benefits. Then, we propose a possible solution based on the Shapley value for a two-player game [48].

6.3.1 Evaluation of coalitional benefit

As mentioned earlier, the structure of each agent's MPC controller is allowed here to evolve according to the time-variant coupling conditions of the system. Such evolution is the outcome of a bargaining procedure over the formation of a coalition between two players. The bargaining is based on an index accounting for both control performance and cooperation-related costs:

$$J_i = \sum_{t=0}^{N_r} \ell_i(t|k) + N_r \chi_i(t), \quad i \in \{1, 2, 1{\cup}2\}, \tag{22}$$

where $\chi_i(t) = f(|\mathcal{C}_i|, |\mathcal{E}_i(t)|)$ expresses the cooperation costs as defined in Section 3.4. The quadratic stage cost ℓ_i, associated with either the merger or the individual players, is evaluated over a time slot of length N_r .

$$\ell_i = \xi_i(k)^\intercal \mathbf{Q}_i \xi_i(k) + \nu_i(k)^\intercal \mathbf{R}_i \nu_i(k), \tag{23}$$

where $\mathbf{Q}_i = \text{diag}(\{Q_j\}_{j\in\mathcal{P}_i})$, $\mathbf{R}_i = \text{diag}(\{R_j\}_{j\in\mathcal{P}_i})$ for $i \in \{1, 2\}$, whereas for the merger

$$\mathbf{Q}_{1\cup2} = \begin{bmatrix} \mathbf{Q}_1 & 0 \\ 0 & \mathbf{Q}_2 \end{bmatrix}, \quad \mathbf{R}_{1\cup2} = \begin{bmatrix} \mathbf{R}_1 & 0 \\ 0 & \mathbf{R}_2 \end{bmatrix}.$$

Notice that in (22), for $i \in \{1, 2\}$ the costs expressed by χ_i involve only player i internal communication (see 3.4 for further details).

To predict the result of either agreement or disagreement on the formation of the new coalition, the MPC problem (24a) is solved for the three possible cost functions associated to $i \in \{1, 2, 1{\cup}2\}$.

$$\min_{\boldsymbol{\nu}_i} \sum_{t=0}^{N_p-1} \boldsymbol{\ell}_i(t|k) + \xi(N_p|k)^\intercal \mathbf{P}_i \xi(N_p|k), \tag{24a}$$

where $\boldsymbol{\ell}_i(t|k)$ is the estimated stage cost (see Remark 3), based on the knowledge available within either set of agents $\mathcal{P}_i$ if $i \in \{1, 2\}$ or, in case of merger, the joint knowledge provided by $\mathcal{P}_1 \cup \mathcal{P}_2$. The symmetric positive definite matrix $\mathbf{P}_i$ weights the last step of the prediction horizon of length N_p. The constraints

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for (24a) are:

$$\xi_i(t+1|k) = \mathbb{A}_i\xi_i(t|k) + \mathbb{B}_i\nu_i(t) + \mathbb{D}_i\omega_i(t|k), \tag{24b}$$

$$\xi_i(t|k) \in \Xi_i,\ t = 0,\dots,N_p, \tag{24c}$$

$$\nu_i(t|k) \in \Psi_i,\ t = 0,\dots,N_p-1, \tag{24d}$$

$$\xi_i(0|k) \equiv \xi_i(k), \tag{24e}$$

$$\omega_i(t|k) = \hat{\omega}_i(k+t),\ t = 0,\dots,N_p-1, \tag{24f}$$

for $i \in \{1, 2, 1{\cup}2\}$. The state transition matrices for the individual players are defined as $\mathbb{A}_1 = [A_{ij}]$ and $\mathbb{B}_1 = [B_{ij}]$ for all $i \in \mathcal{P}_1$ and $j \in \mathcal{P}_1 \cap \mathcal{M}_i$ (otherwise, the corresponding entries in $\mathbb{A}_1$ are null). Matrices for player 2 are composed likewise. For the case $i = 1{\cup}2$, the state transition matrices in (24b) are

$$\mathbb{A}_{1\cup2} = \begin{bmatrix} \mathbb{A}_1 & \mathbb{A}_{12} \\ \mathbb{A}_{21} & \mathbb{A}_2 \end{bmatrix}, \quad \mathbb{B}_{1\cup2} = \begin{bmatrix} \mathbb{B}_1 & \mathbb{B}_{12} \\ \mathbb{B}_{21} & \mathbb{B}_2 \end{bmatrix},$$

where $\mathbb{A}_{12} = [A_{ij}]$ and $\mathbb{B}_{12} = [B_{ij}]$ for all $i \in \mathcal{P}_1$ and $j \in \mathcal{P}_2 \cap \mathcal{M}_i$. A similar definition holds for $\mathbb{A}_{21}$ and $\mathbb{B}_{21}$.

As defined in (5a) and (5b), the term $\omega_i \triangleq \{w_j\}_{j\in\mathcal{P}_i}$ in (24b) accounts for the estimated effect of the coupled subsystems *not involved* in the optimization, i.e., every $r \in \mathcal{M}_{-i}$ where

$$\mathcal{M}_{-i} = \{r|\, r \in (\bigcup_{j\in\mathcal{P}_i} \mathcal{M}_j) \setminus \mathcal{P}_i\}, \tag{25}$$

for each $i \in \{1, 2, 1{\cup}2\}$. So, $\mathbb{D}_1 = [D_{ij}]$ for all $i \in \mathcal{P}_1$ and $j \in \mathcal{M}_{-1}$. A similar definition holds for player 2. The constraint sets are defined by the Cartesian product of the local constraints of the subsystems constituting the player, i.e.,

$$\Xi_i = \Pi_{j\in\mathcal{P}_i}\mathcal{X}_j, \tag{26a}$$

$$\Psi_i = \Pi_{j\in\mathcal{P}_i}\mathcal{U}_j, \tag{26b}$$

$$\tag{26c}$$

6.3.2 Joint benefit through cooperation

Necessary condition for the coalition $\mathcal{P}_1 \cup \mathcal{P}_2$ to form is that its predicted cost (22) outperforms the aggregate cost resulting from unilateral strategies:

$$J^*_{1\cup2} \leq J^*_1 + J^*_2, \tag{27}$$

where the superscript '$*$' designates the values of (22) corresponding to the minimizing input sequences obtained as solution of (24a) for each $i \in \{1, 2, 1{\cup}2\}$. Notice that (27) constitutes the foundation of cooperative MPC algorithms (see, e.g., [49]), where local agents optimize an index that considers the global plant performance.

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Remark 4 *Any new coalition will be product of the union of two players,* and thus of all agents they involve*. The present approach is based on the performance of the player as a whole and not on that of its individual components. One basic advantage of such approximation is that of avoiding the combinatorial explosion of the possible configurations that would arise otherwise.*

6.3.3 Individual rationality

The approximation specified in Remark 4 implies the assumption that if an agreement is beneficial for the entire coalition then it is beneficial for each of its members too. Since the premise here is that the point of view of each single agent is based on its individual cost (20) (agents are rational and selfish), it is essential to check this condition. Here we test this condition on the coarsest scale, i.e., over each pair of bargaining players. Let us first define

$$J_{1\cup 2}^{(j)} \triangleq \sum_{t=k}^{k+N_r} \ell_{1\cup 2}^{(j)}(t) + \chi_{1\cup 2}^{(j)},\ j \in \{1,2\}, \tag{28}$$

as the quota relative to player j in the merger cost $J_{1\cup 2}$. The stage cost $\ell_{1\cup 2}^{(j)}(t)$ employed in (28) is the component of (23) relative to one of the players, for the case in which the merger is produced (i.e., relative to the state and input trajectories obtained by solving the MPC problem (24a) for $i = 1\cup 2$). The value of $\chi_{1\cup 2}^{(j)}$ is a proper share of the cooperation costs.

It can be verified (by assuming, for simplicity, $\chi = 0$) that

$$J_{1\cup 2} \leq J_1 + J_2 \;\not\Rightarrow\; J_{1\cup 2}^{(j)} \leq J_j,\ \forall j \in \{1,2\}. \tag{29}$$

Indeed, the condition (27) does not guarantee *individual* benefit to both players—unless some means of transferring the value between them is provided. On the grounds of individual rationality, a new coalition is formed if and only if a secure benefit can be granted to their future members, i.e., the individual player's payoff obtained through the merger has to be equal to (if not better than) the one obtained through a unilateral strategy. In particular, player j will accept participating in the merger $\mathcal{P}_1 \cup \mathcal{P}_2$ if and only if the following individual rationality requirement is fulfilled:

$$J_{1\cup 2}^{(j)} \leq J_j,\ j \in \{1,2\}. \tag{30}$$

6.3.4 Coalitional TU algorithm

Algorithms based on the sole improvement of the joint benefit, or on individual rationality concerns, are indeed best motivated when the enhancement of performance achieved through the coalition cannot be translated into economical units and, most importantly, cannot be transferred (as recompense) from one player to the other. In the remainder we assume instead that the index (22) can be economically measured. Then, we consider the possibilities opened whenever a value equivalent to the benefit achieved through the merger, i.e.,

$$\Pi = J_1 + J_2 - J_{1\cup 2}, \tag{31}$$

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can be transferred between the players; in other words, we consider a transferable utility (TU) scenario [50]. In such a scenario it is possible to fulfill condition (30) for both players by means of a proper *a posteriori* redistribution of the utility between the players. Notice that here the redistribution of the benefit is equivalent to reallocating the control costs.

If rational players come to an agreement, they will agree on achieving the largest possible payoff. Such joint agreement is referred to as *cooperative strategy*. A similar agreement will belong to the Pareto front (where no allocation can make a player better off without making the other player worse off). The payoffs achieved with unilateral strategies, i.e., (J_1, J_2), constitute the *disagreement point*: a player would not accept a payoff smaller than its own disagreement point.

We see that the Shapley value, due to its inherent properties [48], appears as a clear candidate to compute such allocation. In particular, the most relevant to our scope is the *carrier* axiom [32]: it implies that the benefit is allocated among the players actually contributing to the performance improvement of the merger. More formally, any coalition $\mathcal{C}$ for which it holds that $v(\mathcal{C}') = v(\mathcal{C}' \cap \mathcal{C})$, for any other coalition $\mathcal{C}'$ in the set of players, is referred to as the *carrier* of the game. According to the carrier axiom, the value of the game is confined within the carrier coalition. This implies *(i) efficiency* of the allocation, i.e., the sum of the payoffs assigned to each player in the game equals the aggregate benefit, and *(ii) dummy players* receive a null payoff, i.e., the benefit is distributed only among the players contributing to the value of the coalition.[9] Moreover, individual rationality is guaranteed by the fulfillment of the *superadditivity* condition described by (27). Hence, the allocation provided by the Shapley value qualifies as an *imputation*, i.e., an allocation which is both efficient and individually rational. Notice that for two-player TU games any imputation is in the *core*.[10] This means that such imputation can provide stability to the merger.

The Shapley value for a two-player TU game can be explicitly written as:

$$\phi_i = \frac{1}{2} v(\mathcal{P}_i) + \frac{1}{2} \left[v(\mathcal{P}_1 \cup \mathcal{P}_2) - v(\mathcal{P}_j) \right], \tag{32}$$

for each player $i \in \{1, 2\}$, and $j \in \{1, 2\} \setminus \{i\}$. In order to obtain a fair measure of the cost that each player should incur—inversely related to the *importance* of either player in the merger—we start by defining the coalition values to be employed in (32) as in (33).

$$\begin{aligned} v(\mathcal{P}_1) &= J_1, \\ v(\mathcal{P}_2) &= J_2, \\ v(\mathcal{P}_1 \cup \mathcal{P}_2) &= J_{1\cup 2}. \end{aligned} \tag{33}$$

Following the definitions in (33), the payoffs computed through (32) take the form

$$\phi_i = \frac{1}{2} J_i + \frac{1}{2} (J_{1\cup 2} - J_j), \quad j \in \{1, 2\} \setminus \{i\}, \tag{34}$$

[9] In our scenario, dummy players will be those who do not show any dynamic coupling with the (agents within the) other player).

[10] The reader is referred to [1] for a brief introduction to the basic notions of game theory.

expressing the quota relative to each player in the compound cost index $J_{1\cup 2}$, according to the intrinsic fairness of the Shapley value.

Provided that the necessary condition for coalition formation (27) is fulfilled, consider the general case in which the cost associated to the unilateral strategy of one of the players (designated from now on as player 1) shows a greater associated cost (w.r.t. the other player), i.e.,

$$J_{1\cup 2} \leq J_1 + J_2, \; J_1 > J_2. \tag{35}$$

Now, rearranging (34) as

$$\phi_i = \frac{1}{2} J_{1\cup 2} + \frac{1}{2} \left(J_i - J_j \right), \tag{36}$$

it is easy to see that

$$J_1 > J_2 \implies \phi_1 > \phi_2.$$

For the two-player case, the Shapley value assigns a cost which is a function of the difference of the solitary strategies of the players, centered about one half of the global merger cost.

Now, consider the quantity $\phi_i - J^{(i)}_{1\cup 2}$, i.e., the gap between the *fair* cost defined by the Shapley value for player i, and the cost he actually incurs—the quota $J^{(i)}_{1\cup 2}$ of the merger performance index. Recall, by the efficiency axiom, that $\sum_i \phi_i = J_{1\cup 2}$. Also, $\sum_i J^{(i)}_{1\cup 2} = J_{1\cup 2}$. By subtracting these two equalities, we have

$$\phi_1 - J^{(1)}_{1\cup 2} = -\left[\phi_2 - J^{(2)}_{1\cup 2} \right],$$

revealing that one of the players is excessively benefited—w.r.t. Shapley's distribution of welfare—following its participation in the coalition, whereas the other player experiences the opposite situation. Thus, we can define a unique value for the gap $\epsilon \triangleq |\phi_1 - J^{(1)}_{1\cup 2}|$.

By the transferable utility assumption, we aim to compensate this gap in order to incentivize the formation of a coalition between the players. The cost distribution dictated by (32) can be established by transferring a value $\tau_{\text{TU}} \equiv \epsilon$ from the advantaged player to the other

$$J^{(1)}_{1\cup 2} \pm \tau_{\text{TU}} = \phi_1, \tag{37a}$$

$$J^{(2)}_{1\cup 2} \mp \tau_{\text{TU}} = \phi_2. \tag{37b}$$

To conclude, it is important to recall the situation defined by (29). Notice that, if (27) is strictly satisfied, the allocation computed through (32) always fulfills the following condition:

$$\phi_i < J_i, \quad \forall i \in \{1, 2\} \tag{38}$$

resulting by the individual rationality axiom of the Shapley value. This can be clearly understood by presenting the allocation of costs expressed by ϕ_1 and ϕ_2 from a different perspective. More specifically, we want to show how such allocation relates to the benefit Π yielded by the merger. Taking into account (31), (32) can be rewritten as in (39) in order to explicitly show such relation:

$$\phi_i = J_i - \frac{1}{2}\Pi. \tag{39}$$

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Thus, the Shapley value inherently allocates an equal share of the benefit obtained by the merger to each player, and the final individual payoffs ϕ_i will depend on the costs associated with the solitary strategies. Hence, whenever the *surplus produced by merger is positive*, individually rational players will *always accept to participate in it*, provided that utility transfers are allowed.

6.3.5 Bargaining procedure

Following the criteria detailed above, all players *in pairs* will perform—at given time intervals, in general multiple of the sampling time—a one-shot bargaining, whose outcome will decide the generation of new coalitions. At each round, all available pairs of players have first to be identified. Notice that the sequence with which the pairs are composed may influence the final outcome of the coalition formation process [51]. Then, each player threatens the other with the unilateral strategy it will take if an agreement is not reached. A threat must not hurt the player who makes it to a greater degree than its opponent (otherwise it would not be credible).

Assumption 5 *Since multiple pairs simultaneously carry on their bargaining, there is no way for a pair of players to be aware of the eventual agreements taking place among the rest of players. Thus, when evaluating the possible formation of coalition $\mathcal{P}_1 \cup \mathcal{P}_2$, each pair assumes that the rest of the agents, i.e., $\mathcal{N} \setminus (\mathcal{P}_1 \cup \mathcal{P}_2)$ remain organized as they were at the previous time step.*

Finally, any pair of players verifies (27) and (30) (only if the TU assumption does not hold) before stipulating the agreement, following Assumption 5 in the computation of (24a). This means that eventual changes of configuration concerning subsystems external to $\mathcal{P}_1 \cup \mathcal{P}_2$ is not taken into account on the estimate of the unknown coupling $\hat{\omega}_i$.

6.4 The study of price mechanisms from WP2 for coalition formation[11]

Let $\mathcal{P} = \{\mathcal{C}_i\}$, $i = 1, \dots, N_c$, denote the set of all coalitions analysed in Subsection 6.3 (recall that $\mathcal{P}$ is a partition of $\mathcal{N}$). After calculation performed in Subsection 6.3 one can create a graph $\mathcal{G}' = (\mathcal{P}, \mathcal{L})$ for which the set of vertices comprises all considered coalitions, while $\mathcal{L}$ contains only those edges between two vertices (coalitions) for which we have computed a positive benefit of the merger, i.e., $(i, j) \in \mathcal{L}$ if we have computed $v(\mathcal{C}_i \cup \mathcal{C}_j)$ and $v(\mathcal{C}_i) + v(\mathcal{C}_j) - v(\mathcal{C}_i \cup \mathcal{C}_j) > 0$. Furthermore, let $\mathcal{W} = \{w_{i,j} \in \mathbb{R} \mid (i, j) \in \mathcal{L}\}$ denote the set of weights for all edges, with $w_{i,j}$ being equal to the value of the benefit of the merger:

$$w_{i,j} := v(\mathcal{C}_i) + v(\mathcal{C}_j) - v(\mathcal{C}_i \cup \mathcal{C}_j) = J_i + J_j - J_{i \cup j}, \quad (i, j) \in \mathcal{L}. \tag{40}$$

A greedy heuristic approach to coalition formation merges only two coalitions $\mathcal{C}_{i^*}$ and $\mathcal{C}_{j^*}$ that achieve the largest benefit $(i^*, j^*) = \arg\max_{(i,j)} w_{i,j}$. However, as pointed out in [52], one can use a more advanced strategy to speed up the merging process by treating all possible mergers simultaneously. We

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can find this set of combinations by solving a matching problem. For a graph $\mathcal{G}' = (\mathcal{P}, \mathcal{L})$, a subset $\mathcal{M}$ of $\mathcal{L}$ such that no two edges in $\mathcal{M}$ are incident to a common node is called a matching. We allow nodes to remain unmatched and have to solve a maximum weight matching in the graph $\mathcal{G}'$ to obtain the set of combinations producing the greatest benefit

$$\begin{array}{rcll} \mathcal{M}^* & = & \arg\max_{\mathcal{M}} & \sum_{(i,j)\in\mathcal{M}} w_{i,j} \\ & & \text{s.t} & \mathcal{M} \text{ matching } \mathcal{L}. \end{array} \tag{41}$$

The maximum weight matching problem is well studied in the literature, with reported polynomial-time algorithms, for more details see [52, 53] and references therein.

In [54] it is pointed out that the dual of the planar graph for the Minimum Spanning Tree (MST) problem can be interpreted as a bipartite maximum weight perfect matching problem. In that context, the parametric analysis of the solution of the MST problem, as carried out in Chapter 6 of DYMASOS deliverable D2.2 "Report on algorithms for dynamic reconfiguration applied to economics based coordination in SoS", could be utilized to analyse formation of coalitions.

7 Minimization of mutual disturbance

The control agents can coordinate their action in order to reduce the effects of coupling between subsystems. This section presents a control scheme based on a robust tube MPC (see, e.g., [55]), requiring exchange of information about the sets where the state and input trajectories will be confined. The robust tubes are optimized online following the dynamics of the subsystems' coupling. More specifically, with the objective of reducing the conservativeness shown by previously published works on dynamically coupled systems, the state and input trajectories are confined in the minimal sets allowing to fulfill the control goals, which translates in minimizing the size of the considered disturbance sets. For further details, the reader is referred to [56].

7.1 Problem definition

Consider a the same system described in Section 3.1, composed of a set $\mathcal{N} = \{1, \ldots, N\}$ of subsystems whose individual state trajectory evolves according to (1):

$$x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + w_i(k),$$

Coupling between subsystems is described by (2):

$$w_i(k) = \sum_{j \in \mathcal{M}_i} (A_{ij}x_j + B_{ij}u_j),$$

where the *set of neighbors* $\mathcal{M}_i$ identifies all the subsystems $j \in \mathcal{N} \setminus \{i\}$ showing some dynamical effect on subsystem i. The evolution of the state and input trajectories is constrained within the polytopic sets $\mathcal{X}_i$ and $\mathcal{U}_i$, respectively. Hence, the disturbance induced by neighboring subsystems to subsystem i is confined within

$$w_i \in \mathcal{W}_i = \bigoplus_{j \in \mathcal{M}_i} (A_{ij}\mathcal{X}_j \oplus B_{ij}\mathcal{U}_j). \tag{42}$$

Now, assume that for each subsystem there exists a local linear feedback law K_i such that $(A_{ii} + B_{ii}K_i)$ is stable. Furthermore, assume that each mutual disturbance set $\mathcal{W}_i$ is a polytope containing the origin. Then, for each $i \in \mathcal{N}$, the evolution of each subsystem can be bounded within a given set, i.e., there exists a robust positively invariant set $\mathcal{R}_i$ such that

$$(A_{ii} + B_{ii}K_i)\mathcal{R}_i \oplus \mathcal{W}_i \subseteq \mathcal{R}_i. \tag{43}$$

7.2 Control policy

The optimal control problem for mutual disturbance minimization is formulated next. The final objective is that of driving the subsystem to the nominal behavior, i.e., *where disturbance from neighbors is absent*, in order to minimize a quadratic cost expressed by $J = \sum_{i \in \mathcal{N}} J_i$, where

$$J_i \triangleq \sum_{t=0}^{N_p-1} \frac{1}{2} x_i^\intercal(t|k)Q_i x_i(t|k) + u_i(t|k)^\intercal R_i u_i(t|k) + x_i^\intercal(N_p|k)P_i x_i(N_p|k)$$

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is the cost associated to each individual subsystem over a N_p-step time window. The state and input vectors z_i and v_i define the *nominal* dynamics:

$$z_i(k+1) = A_{ii}z_i(k) + B_{ii}v_i(k). \tag{44}$$

Then, each subsystem is fed with a composite control action,

$$u_i = v_i + K_i(x_i - z_i), \tag{45}$$

where v_i is obtained by the solution of the following MPC control problem, employing the nominal model (44),

$$\min_{\mathbf{v}_i} J_i(z_i, v_i) + \rho \left\| a_i + b_i \right\|, \tag{46a}$$

s.t.

$$z_i(t+1|k) = A_{ii}z_i(t|k) + B_{ii}v_i(t|k), \tag{46b}$$

$$z_i(t|k) \in \mathcal{X}_i(a_i) \ominus \mathcal{R}_i,\ t = 0, \ldots, N_p, \tag{46c}$$

$$v_i(t|k) \in \mathcal{U}_i(b_i) \ominus K_i\mathcal{R}_i,\ t = 0, \ldots, N_p - 1, \tag{46d}$$

where $a_i = \{a_{ir}\} \in \mathbb{R}^{r_x}$, where r_x is the number of inequalities defining the polytope $\mathcal{X}_i$, and $a_{ir} \in [0, 1]$ (an analogous definition holds for b_i). Through the parametrization of the state and input constraints (46c) and (46d), the role of the second term in the objective function (46a) is that of minimizing the size of the set where the evolution of the state of subsystem i is allowed. Since the disturbance affecting each subsystem is given by (42), by applying such control policy over all subsystems, the mutual disturbance can be minimized.

For a deeper discussion, the reader is referred to [56].

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8 Distributed coalitions in population control approaches[12]

8.1 Motivating application: demand response management in smart grids

Due to the introduction of distributed and unpredictable energy sources, as for example renewables, into the power grid, the topic of energy management is becoming crucial in power systems. To accommodate for the increased uncertainty in energy supply, it has recently been proposed to match energy demand and supply by regulating the load consumption, together with the energy production. Matching demand and supply can be achieved via the so-called demand response methods, such as *direct control* and *real-time pricing*. While in the former the energy providers have the authority to switch on and off the controlled loads, in the latter the users keep their control authority but are subject to population incentives, such as variable electricity prices, usually proportional to the instantaneous total energy demand. The latter scenario has been recently analyzed using noncooperative game theory and convex optimization [57, 58]. Among other applications, demand response methods have been studied to compute optimal charging strategies for large populations of Plug-in Electrical Vehicles (PEVs) [59, 60].

Following the lines of [61], the effect of demand response methods can be modeled by assuming that each agent $i \in \mathcal{N} = \{1, \dots, N\}$ schedules its demand $\mathbf{d}_i = [d_i(1), \dots, d_i(T)] \in \mathbb{R}^T$ over the horizon $\mathcal{T} = \{1, 2, \dots, T\}$ by solving the following optimization problem

$$\begin{aligned} \mathbf{d}_i^\star(\mathcal{A}) \triangleq \arg \min_{\mathbf{d}_i \in \mathbb{R}^T} \; & \theta \|\mathbf{d}_i - \hat{\mathbf{d}}_i\|^2 + (\lambda \mathcal{A} + p_0)^\intercal \mathbf{d}_i \\ \text{s.t.} \quad & \mathbf{d}_i \in \mathcal{D}_i \end{aligned} \tag{47}$$

where $\mathcal{A} = \frac{1}{N} \sum_{i=1}^N \mathbf{d}_i \in \mathbb{R}^T$ is the vector of the population aggregate demand for the time interval $\mathcal{T}$, and the set $\mathcal{D}_i$ models physical constraints, such as demand bounds and rates, and user preferencies, including intertemporal constraints. The first term in the cost function in (47) models the curtailment cost that each agent encounters for deviating from its nominal energy schedule $\hat{\mathbf{d}}_i$, according to the Taguchi loss function [62], where $\theta > 0$ is a constant conversion coefficient. The second term models the price that each agent has to pay for the required energy according to an affine price function $p(\mathcal{A}) := \lambda \mathcal{A} + p_0$, where $\lambda > 0$ is a parameter related to the elastic pricing and p_0 is the basic, possibly time-varying, price for unitary energy consumption. Since the price function depends on the aggregate strategy of all the players, this problem can be addressed via deterministic mean field game formulations [63–65].

In the literature, the incentive $p(\mathcal{A})$ is typically broadcast to the users by a central operator that computes ahead of time the aggregate consumption of the loads over the whole time horizon $\mathcal{T}$ and updates the incentive accordingly, see [57, Figure 2]. In the setup described above, the individual agents are selfish, in the sense that they do not cooperate. Clearly, it is possible that the agents could benefit from local interactions and cooperations among other (neighbouring) agents, which motivates the analysis of coalitions of agents.

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8.2 Distributed coalitions in population control approaches

In this section, we consider a general scenario where the agents are grouped in coalitions via a communication network, so that they have the chance to cooperate locally. Specifically, we consider a quasi-aggregative game played between the agents of the population, and we assume that the N agents communicate according to a row stochastic matrix $P \in \mathbb{R}^{N \times N}$, whose element $P_{ij} \in [0, 1]$ indicates the strength (or relevance) of the communication from agent j to agent i, where $P_{ij} = 0$ denotes no communication from agent j to i, and the diagonal elements are set to zero, that is $P_{ii} = 0$ for all i. The structure of the matrix P determines the *local coalitions* of agents. In the following, we denote by $\mathcal{M}_i$ the set of neighbors of agent i, that is $\mathcal{M}_i := \{j \in \{1, \ldots, N\} \mid P_{ij} > 0\}$. Note that we consider j to be a neighbor of agent i if agent i *receives* communications from j. Moreover, we assume that the interaction between neighbors is not one-to-one, but each agent i is influenced only by the aggregate strategies of its neighbors $\mathcal{M}_i$. More in detail, each agent i tries to minimize a cost function $J_i(x_i, \sigma_i)$ that depends on its own deterministic state $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ and on the ith *coalition state*

$$\sigma_i \triangleq \sum_{j \neq i} P_{ij} x_j \in \mathbb{R}^{n_i}. \tag{48}$$

Formally, each agent $i \in \{1, 2, \ldots, N\}$ aims at solving the optimization problem

$$x_i^\star(\sigma_i) = \arg \min_{x_i \in \mathcal{X}_i} J_i(x_i, \sigma^i). \tag{49}$$

Let us consider the class of aggregative games with heterogeneous convex compact constraints $\mathcal{X}_i$ and quadratic cost

$$J_i(x_i, \sigma^i) \triangleq q_i x_i^\intercal Q x_i + 2 \left(C_i \sigma^i + c_i\right)^\intercal x_i, \tag{50}$$

where $q_i > 0$, Q, $C_i \in \mathbb{R}^{n_i \times n_i}$, $Q \succ 0$, $c_i \in \mathbb{R}^{n_i}$.

Since the agents have interest in optimizing their own cost functions J_i given the aggregate information from the local coalition, the following distributed control algorithm has been proposed in [66].

Initialization: Set $k = 0$. Fix $\lambda \in (0, 1)$. Each agent i computes the initial coalition state $\sigma_i(0) \triangleq \sum_{j \neq i} P_{ij} x_j(0)$ and sets $z_i(0) = \sigma_i(0)$.

Iteration: Each agent i computes its optimal strategy given the internal state $z_i(k)$:

$$x_i^\star(k+1) = \arg \min_{x_i \in \mathcal{X}_i} J_i(x_i, z_i(k));$$

each agent i updates the coalition state: $\sigma_i(k+1) = \sum_{j \neq i} P_{ij} x_j^\star(k+1)$;

and updates its internal state: $z_i(k+1) = \lambda z_i(k) + (1 - \lambda) \sigma_i(k+1)$;

$k \leftarrow k + 1$.

Technical conditions on the cost parameters (q_i, Q, C_i) under which the above algorithm is guaranteed to converge to a Nash equilibrium for the population of agents are established in [66].

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8.3 Application to coalitions of smart buildings

The setting introduced in (47) can be also used to analyse coalitions of smart building populations, specifically for heating ventilation air conditioning (HVAC) systems. As suggested in [61], suppose that each smart building schedules optimally its HVAC usage as follows:

$$\begin{aligned} x_i^\star(\mathcal{A}) &\triangleq \arg\min_{x_i\in\mathbb{R}^T} \theta\gamma^2 \left\|x_i - \hat{x}_i\right\|^2 + (\lambda\mathcal{A} + p_0)^\intercal x_i \\ \text{s.t.} \quad & x_i \in \left[x_i^{\min}, x_i^{\max}\right] \end{aligned} \tag{51}$$

where $\theta > 0$ is the cost coefficient of the Taguchi loss function, $\gamma > 0$ specifies the thermal characteristic of the HVAC system, $\lambda > 0$ is the elastic price constant and p_0 is the basic price for unit of energy consumption. In [61, Theorems 1, 2] it is shown that if

$$\lambda \leq \frac{2\theta\gamma^2}{N-3} \tag{52}$$

then the distributed control algorithm in [61, Equations 8, 9] can be used to compute Nash equilibrium for the problem in (51). Note that the set of elastic prices λ such that (52) holds shrinks linearly as the population size decreases. On the other hand, Algorithm 2 in [66] guarantees convergence to an almost Nash equilibrium for the more general case of convex constraints $x_i \in \mathcal{X}_i$ instead of box constraints, under the less stringent assumption $\lambda < 2\theta\gamma^2$ which is independent of the population size N, via a communication network P such that $\|P\|_2 \leq 1$.

As a particular case, we next consider a hierarchical communication structure where a total of N buildings are grouped in M coalitions. For simplicity of notation, let us assume that each coalition comprises B buildings. At every communication step, the coalition managers compute the aggregate power demand of their buildings, then communicate among each other using a communication matrix P_M and finally compute the incentive signal for their coalitions. With the convention that buildings controlled by the same manager are grouped together in the extended vector and that the manager is the first agent of the corresponding block, the above coalition management scheme corresponds to a communication matrix

$$P \equiv \underbrace{\left(I_M \otimes \begin{bmatrix} 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}\right)}_{\text{incentives}} \underbrace{(P_M \otimes I_B)}_{\text{communication}} \underbrace{\left(I_M \otimes \begin{bmatrix} 1/B & \cdots & 1/B \\ 0 & \dots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \dots & 0 \end{bmatrix}\right)}_{\text{coalition aggregation}} = P_M \otimes \frac{1}{B}\mathbb{1}_B\mathbb{1}_B^\intercal. \tag{53}$$

It is possible to show that if P_M satisfies basic connectivity conditions, then also the matrix P in (53) does, and that $\|P_M\|_2 \leq 1$ implies $\|P\|_2 \leq 1$. Therefore, it follows from [66] that the iterative algorithm proposed above can be used to drive the smart buildings of the coalitions towards a Nash equilibrium solution in a distributed fashion.

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9 Information-aware coalition formation

Information plays a fundamental role in the coordination of multiagent systems [67]. Either attained through direct sensing or communication, information is the basis for the local decisions of agents and, as such, decisive to the emergent global behaviour [67]. A fundamental problem in the inherent distributed decision-making framework associated to coalitional control is how the information flow impacts the achievable performance. Such connection is still largely unexplored [68].

Furthermore, as the architecture of multiagent systems becomes more complex, featuring reconfigurable topologies (see, e.g., [19, 22, 69, 70]), misaligned individual interests (e.g., [26, 28, 45, 71, 72], human-in-the-loop control (e.g., [25]), the relationship between possible restrictions on the global availability of information and the system performance is increasingly unclear [67]. The need for locality emerges from the vastness of the mission space, coupled with communication and sensing limitations, that hinder local controllers from having global knowledge [68].

Does providing agents with additional information always lead to improvements in the performance? On the other hand, can the excess of information be detrimental under some given system-wide perspectives?

The authors of [67] address the questions above. As a simple platform for studying the effect of information on multiagent collaboration, a graph-coloring problem is considered in [67]. In such setting, the authors show the impact of the degree of information into the efficiency of the stable coloring profile—a Nash equilibrium—and into its associated convergence rates. Results demonstrate that an increased amount of information is able to improve the efficiency of the Nash equilibria; on the other hand, it degrades the convergence rate of the distributed color adjustment process, where the agents seek to maximize their utility.

For spatially distributed sensing agents, the work of [73] identifies how a measure of locality in the individual control laws can be translated into a bound on the overall achievable efficiency. In particular, the author of [73] addresses the problem of allocating a collection of agents across a mission space,[13] in order to optimize a given submodular global objective. The relationship between the redundancy of information in the control laws implemented by agents and the achievable efficiency of the overall behaviour is then characterized (bounds are given on the efficiency of the stable solutions). When full information regarding the mission space is available to the agents, the efficiency of the resulting stable solutions is guaranteed to lie within 50% of the optimum. However, as the reach of the information becomes more limited, the efficiency of the stable solutions may be as low as $1/N$ of the optimum, where N designates the number of agents. For further details, the reader is referred to [68, 73].

[13]Such problem belongs to the class of networked resource allocation problem.

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9.1 Partial system information

It is natural to expect that the components of a large-scale heterogeneous system show selfish interests, hindering the free sharing of knowledge of local information across the whole system. Indeed, this constitutes an issue when coupling between privacy-concerned subsystems—that likely have incorrect models of the rest of the system—needs to be dealt with, and non-local information is critical for adequate control feedback. Although in some cases local measurements and input sequences are treated as private information, most distributed control schemes developed over the last decade assume public knowledge of the global system model [17]. Concerning the information exchange in the control of networked systems, researchers mainly focused on the transmission medium itself, hence on issues related to limited bandwith, data loss, noisy channel [27]. In relation with coalition formation—and in particular with the case when only a subset of the control agents is willing to exchange information about their subsystems—our interest is on characterizing the performance achievable by a control loop with *partial system information*. On such line of research, recent studies by [15, 17] are available in the literature.

The fundamental objective of coalitional control is to address the controller design problem on a different perspective with respect to the traditional framework, i.e., where the classic closed-loop objective cannot be given a priori. Taking the standpoint clearly expressed by [17], coalitional control aims at addressing problems whose solution begins by accounting for the constraints on the available plant description. When controller designers have to cope with partial models of the plants, one of the questions that naturally arise is [17]: *is it possible to relate the achievable closed-loop performance with the amount of available information?* Such issues have been already studied in the field of economics, employing tools provided by the game theory. The authors of [17] extend a performance metric—the *competitive ratio*—introduced by [74] for the purpose of quantifying the distance from the optimum of the distributed solution of an linear programming problem when the information locally available is segmented.

Notice that the above question can be reversed. Indeed, instead of assuming that the constraints on information availability define the problem *a priori*, the following question can be addressed: *given some cost objective, what is the minimal partition of the global system model information necessary to guarantee some performance goal?* In order to address this question, the authors of [17] point out that additional metrics need to be formulated, to allow the characterization of the different partition possibilities, and the related *minimality* notion. Again, possible candidates for such task are already available in the game theory literature, related to multi-agent decision making under partial information [17].

The sharing of information in a networked system can be represented through a *knowledge graph*, where each node stands for the model of a given subsystem, and the edges indicate that the information about the models is available to the agents controlling the pointed nodes. *Connectivity* and the *degree distribution* of such graph can be interpreted in quantitative terms [17]. The following definition of knowledge graph is given in [17].

Definition 6 (Knowledge graph) *Let $\Delta : \mathcal{P} \to \mathbb{R}^{n\times n}$ be a feedback law design method. The* knowl-

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edge graph *associated to* Δ *is a directed graph* $\mathcal{G} = (\mathcal{N}, \mathcal{E})$*, with* $\mathcal{N} = \{1, \ldots, n\}$*, and the set of links defined as*

$$\mathcal{E} = \{(i,j) | \, \exists \mathbf{P} = (A, x) \in \mathcal{P} \text{ such that } [\Delta(\mathbf{P})]_{i,\cdot} = f(A_{j,\cdot})\},$$

where $A_{j,\cdot}$ *denotes the* j*th row of matrix* A*, and* $[\Delta(\mathbf{P})]_{i,\cdot}$ *the* i*th row of the feedback law synthesized through the map* Δ*.*

9.2 Coalition formation based on PageRank

In this section, the problem of node aggregation in a graph modeling the interactions among the actors of a SoS is addressed [75]. A prior task that needs to be performed for this purpose is the computation of the value of each node, through the employment of given *measures* of relevance. The measures typically employed in network (game) theory are formulated on the basis of the inherent structure of the network; thus, they seldom find a straightforward application in the control of dynamic networked systems. Nevertheless, some proposals specifically related to distributed control have been recently formulated. For example, in [22] the relevance of nodes in a distributed control system is expressed as a function of the cost-to-go of the closed loop system under the different possible network topologies.

The present discussion is based on the PageRank index, a variation of the eigenvector centrality measure (see [75] and references therein for further details). The PageRank index essentially relates the value of a given node to the number of other nodes pointing to it, as well as to their relative importance.

In [76], a coalitional control scheme employing the PageRank index for the characterization of the coalition values is presented. Local control agents are allowed to create links among them with the aim of establishing an *aid network*. The PageRank of the aid network is then used as a means to group the agents. Model predictive control (MPC) is implemented inside each coalition in order to calculate the control actions without a central coordinator.

Furthermore, the work of [76] provides tools for the computation of the relevance of links, from the viewpoint of the nodes they connect. This information can be used to determine whether it is advantageous to incur some cost to be able to include their associated information in the control law. It is worth to point out that measures to determine the relevance of links from their related nodes are already present in works like [77] and have been proposed in the context of control systems for system partitioning in [78]. Likewise, a modified version of PageRank is proposed in [79] to the problem of finding optimal input nodes of multi-agent systems. However, it is worth to point out that in these works such measures are derived offline and used for analytical purposes. The algorithm proposed in [76] computes measures in a distributed fashion, using them as a prescriptive tool for dynamically grouping local control nodes. Further details are given in the remainder.

9.2.1 Computation of the PageRank

This section contains a brief description of the computation of the PageRank value [75, 76]. Consider a network represented as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \dots, n\}$ is the set of vertices or nodes and $\mathcal{E}$ is the set of edges representing the links among the nodes. In case that a node i has a link pointing to node j then $(i, j) \in \mathcal{E}$. Recall that the PageRank value of a node is an eigenvector centrality measure. In particular, each node i is given a value $p_i^* \in [0, 1]$, with $\sum_{i \in \mathcal{N}} p_i^* = 1$. This value is defined by the sum of the contributions from all the nodes pointing to it, i.e.,

$$p_i^* = \sum_{j \in \mathcal{N}_i^+} \frac{p_j^*}{n_j}$$

where $\mathcal{N}_i^+ := \{j : (j, i) \in \mathcal{E}\}$ is the set of nodes pointing to node i and n_j is the number of outgoing links of node j. Hence, PageRank assigns relevance based on the assumption that a node having links from other important nodes must also be relevant. Let $p^* := [p_i^*]_{i \in \mathcal{N}}$ be the vector of the PageRanks of all nodes. The PageRank vector p^* is the nonnegative unit eigenvector that corresponds to the eigenvalue 1 of A. Then the calculation of the PageRank can be stated in matrix form

$$p^* = Ap^*, \quad p^* \in [0, 1]^n, \quad \sum_{i \in \mathcal{N}} p_i^* = 1 \tag{54}$$

where A is the so-called hyperlink matrix, a variation of the adjacency matrix, given by

$$a_{ij} = \begin{cases} \frac{1}{n_j} & j \in \mathcal{N}_i^+, \\ 0 & \text{otherwise.} \end{cases}$$

We assume that A is a stochastic matrix, i.e., it verifies $\sum_{j \in \mathcal{N}} a_{ij} = 1$ for all $j \in \mathcal{N}$. As it is shown in [80], in order to have a graph of the Internet network that satisfies this property it is necessary to remove all nodes having no links to other nodes. In order to guarantee the uniqueness and existence of p^*, (54) is slightly modified and the PageRank vector p^* is defined as the solution of

$$p^* = Mp^*, \quad p^* \in [0, 1]^n, \quad \sum_{i \in \mathcal{N}} p_i^* = 1, \tag{55}$$

where M is a convex combination of A and $\mathbf{1}^{n \times n}$, i.e., $M := (1 - m)A + \frac{m}{n}\mathbf{1}^{n \times n}$, $m \in (0, 1)$.

We finish this section with the introduction of a measure of the relevance of the links of the network. We define the *LinkRank* of an edge (i, j) as the sum of the PageRanks that flow between of nodes i and j, i.e.,

$$l_{ij} = \begin{cases} \frac{p_i^*}{n_i} + \frac{p_j^*}{n_j} & \text{if } (i, j), (j, i) \in \mathcal{E}_i, \\ \frac{p_i^*}{n_i} & \text{if } (i, j) \in \mathcal{E}_i, \\ \frac{p_j^*}{n_j} & \text{if } (j, i) \in \mathcal{E}_i, \\ 0 & \text{otherwise.} \end{cases}$$

Hence, those links that connect nodes characterized by a high PageRank value will be regarded as the most important [76].

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9.2.2 Node aggregation based on PageRank

In [75], the PageRank index associated to a given network node—that can be the result of the aggregation of two or more agents—is proposed as the *characteristic function* assigning a value to each of the possible coalitions. First, define the set of control agents in the system as $\mathcal{N} = \{1, \ldots, N\}$. Initially, the interaction within the set of agents can be modeled as the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{E}$ is such that $(i, j) \in \mathcal{E}$, with $i, j \in \mathcal{N}$, if and only if the agent i interacts with agent j. Once a given set $\mathcal{C} = \{a_1, a_2, \ldots, a_c\} \subseteq \mathcal{N}$ of agents positively assess the benefit of merging into a coalition, the graph is updated with the description of the new configuration of the network, which will be called $\mathcal{G}'$. In $\mathcal{G}'$, the coalition will be designated by a node that preserves all the original links relative to each player in $\mathcal{C}$.[14] For convenience the set $\mathcal{R} = \{r_1, r_2, \ldots, r_r\}$ is also defined as the complimentary set $\mathcal{N} \setminus \mathcal{C}$. The value of the coalition is the PageRank associated to the new node in $\mathcal{G}'$.

The aggregation of nodes can be achieved by means of three auxiliary transformation matrices, namely $\mathbf{P}_\mathcal{C}$, $\mathbf{T}_\mathcal{C}$, and $\mathbf{D}_\mathcal{C}$. Matrix $\mathbf{P}_\mathcal{C}$ is a permutation matrix whose purpose is to rearrange the hyperlink matrix so that the players inside $\mathcal{C}$ appear next to each other in the first columns and rows. In particular, the permutation is given by

$$\mathbf{P}_\mathcal{C} = \left[\mathbf{e}_{a_1}\mathbf{e}_{a_2}\ldots\mathbf{e}_{a_c}\mathbf{e}_{r_1}\mathbf{e}_{r_2}\ldots\mathbf{e}_{r_r}\right],$$

where $\mathbf{e}_i$ denotes a column vector of length $|\mathcal{N}|$ with 1 in the i-th position and 0 in every other position. $\mathbf{T}_\mathcal{C}$ is a transformation matrix whose goal is to aggregate into a single node the members in $\mathcal{C}$. It is given by

$$\mathbf{T}_\mathcal{C} = \begin{bmatrix} \mathbf{1}^{|\mathcal{C}|\times 1} & \mathbf{0}^{|\mathcal{C}|\times|\mathcal{R}|} \\ \mathbf{0}^{|\mathcal{R}|\times 1} & \mathbf{I}^{|\mathcal{R}|\times|\mathcal{R}|} \end{bmatrix}.$$

Finally, $\mathbf{D}_\mathcal{C}$ is a matrix that guarantees that the new hyperlink matrix $\mathbf{A}'$ is also a stochastic matrix. This is done by normalizing the column corresponding to the coalition

$$\mathbf{D}_\mathcal{C} = \begin{bmatrix} \frac{1}{|\mathcal{C}|} & \mathbf{0}^{1\times|\mathcal{R}|} \\ \mathbf{0}^{|\mathcal{R}|\times 1} & \mathbf{I}^{|\mathcal{R}|\times|\mathcal{R}|} \end{bmatrix}.$$

The hyperlink matrix of $\mathcal{G}'$ can be derived from the original network $\mathcal{G}$ as follows

$$\mathbf{A}' = \mathbf{T}_\mathcal{C}^\mathrm{T}\mathbf{P}_\mathcal{C}^\mathrm{T}\mathbf{A}\mathbf{P}_\mathcal{C}\mathbf{T}_\mathcal{C}\mathbf{D}_\mathcal{C}.$$

Consequently, $\mathbf{M}' := (1-m)\mathbf{A}' + m\mathbf{J}'$, with $\mathbf{J}' = \frac{1}{|\mathcal{R}|+1}\mathbf{1}^{|\mathcal{N}'|\times|\mathcal{N}'|}$. As can be seen, $\mathcal{C}$ is mapped into the first vertex of the graph $\mathcal{G}'$.

The value of coalition $\mathcal{C}$ is defined as the PageRank of the associated node in $\mathcal{G}'$, resulting from the aggregation of the relative nodes in $\mathcal{G}$, i.e., the first component of the new PageRank vector $\mathbf{p}'^*$. Hence

$$v(\mathcal{C}) = [1\;0\ldots0]\mathbf{p}'^* = \mathbf{e}_1^\mathrm{T}\mathbf{p}'^*.$$

[14] The links previously existing between the members of the new coalition are converted into self-references for the new node.

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The properties of $\mathbf{M}'$ allow calculating the eigenvector by means of the power method. To this end, let $\mathbf{p}'_0 = \frac{1}{|\mathcal{N}'|}\mathbf{1}^{|\mathcal{N}'|\times\mathbf{1}}$. The PageRank vector can then be calculated as the limit of the sequence generated by

$$\mathbf{p}'[k+1] = M'\mathbf{p}'[k] = (1-m)A'\mathbf{p}'[k] + \frac{m}{|\mathcal{R}|+1}\mathbf{1}^{|\mathcal{N}'|\times 1}$$

when $k \to \infty$, which we denote by $\mathbf{p}'^*$. Finally, the characteristic function of the PageRank aggregation game becomes

$$v(\mathcal{G}) = \mathbf{e}_1^\intercal \mathbf{p}'^*, \tag{56}$$

which provides the PageRank that results from the aggregation of the nodes relative to coalitions in $\mathcal{G}$. Note that (56) depends only on the structure of the network $\mathcal{G}$.

9.2.3 The PageRank difference aggregation game

While knowing which players are expected to provide more PageRank when aggregated into a coalition is valuable information, the net effect of the aggregation still needs to be considered. More specifically, it is interesting to know whether the PageRank of the merge is lower or equal to the sum of the individual PageRanks of the players in the coalition. To this end, we define the PageRank difference aggregation game over the same set of players, with the following characteristic function

$$v_{\text{diff}}(\mathcal{C}) = v(\mathcal{C}) - \sum_{i\in\mathcal{C}} v(i),$$

i.e., the characteristic function $v_{\text{diff}}(\mathcal{C})$ measures the gain or loss of PageRank derived from the aggregation. If $v_{\text{diff}}(\mathcal{C}) > 0$ for a certain coalition $\mathcal{C}$ then the aggregation is rational and the players really have an incentive to perform the coalition. The Shapley value [48] of the difference game provides information about the best players to be aggregated from this perspective. Once the Shapley value $\phi_i(\mathcal{N}, \boldsymbol{v})$ has been computed for each node and all possible coalitions, from (56), the difference aggregation game can be obtained as[15]

$$\phi_i(\mathcal{N}, \boldsymbol{v}_{\text{diff}}) = \phi_i(\mathcal{N}, \boldsymbol{v}) - \boldsymbol{v}(i).$$

To overcome the computational complexity of the Shapley value, a numerical approximation, based on the randomized method in [81], is proposed by [75].

9.2.4 Local approach for node aggregation based on PageRank

Information about all the nodes in the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ may not be available. For such cases, the authors of [75] propose an approximation of $v(\mathcal{C})$ (see (56)) associated to a reduced graph $\mathcal{G}_{\text{sub}}(\mathcal{N}', \mathcal{E}') \in \mathcal{G}$, corresponding to a (local) subset of nodes. A simple heuristic—viable if (sufficiently good) knowledge of the links between the nodes in $\mathcal{N} \setminus \mathcal{N}'$ is available—consists in lumping all nodes in $\mathcal{N} \setminus \mathcal{N}'$ into a single node, and deriving the number of self-references of the resulting combined node. If such knowledge is not available, the following method for the derivation of equivalent subnetworks is proposed in [75].

[15]This is possible by linearity of the Shapley value.

We start by defining the notion of network equivalence employed in [75].

Definition 7 (Graph PageRank equivalence [75]) *Let $\mathcal{G}$ and $\mathcal{G}'$ be two networks containing a subnetwork $\mathcal{G}_{\text{sub}}$. They are* equivalent *in a strict PageRank sense for $\mathcal{G}_{\text{sub}}$ if the PageRank of the nodes in $\mathcal{G}_{\text{sub}}$ is the same in both networks. We denote this by $\mathbf{p}^{\mathcal{G}}(\mathcal{G}_{\text{sub}}) = \mathbf{p}^{\mathcal{G}'}(\mathcal{G}_{\text{sub}})$.*

Hence, the goal is to find a network $\mathcal{G}'$ simpler than $\mathcal{G}$, and equivalent to it in PageRank sense. To this end, the nodes in $\mathcal{G}$ are classified in the following three disjoint categories:

- *external nodes*: a node i is external if $i \in \mathcal{G}$ but $i \notin \mathcal{G}_{\text{sub}}$;
- *mid nodes*: a node i is an mid-node if $i \in \mathcal{G}_{\text{sub}}$ but is linked to a node $j \notin \mathcal{G}_{\text{sub}}$;
- *core nodes*: this group includes all the nodes $i \in \mathcal{G}_{\text{sub}}$ that are linked only to nodes in $\mathcal{G}_{\text{sub}}$.

Let $\mathbf{A}$ and $\mathbf{A}'$ be the hyperlink matrices that corresponds respectively to $\mathcal{G}$ and $\mathcal{G}'$. Without loss of generality, we assume that the elements in $\mathbf{A}'$ are arranged in the following order: external nodes (e), mid nodes (m), and core nodes (c). Thus, $\mathbf{A}'$ has the following structure:

$$\mathbf{A}' = \begin{bmatrix} \mathbf{A}'_{ee} & \mathbf{A}'_{em} & \mathbf{A}'_{ec} \\ \mathbf{A}'_{me} & \mathbf{A}'_{mm} & \mathbf{A}'_{mc} \\ \mathbf{A}'_{ce} & \mathbf{A}'_{cm} & \mathbf{A}'_{cc} \end{bmatrix}. \tag{57}$$

The problem of finding an equivalent network translates in computing an hyperlink matrix $\mathbf{A}'$ according to the following theorem:

Theorem 8 *[75] Let $\mathbf{A}'$ be a nonnegative stochastic matrix described by (57). The networks $\mathcal{G}$ and $\mathcal{G}'$ are equivalent in a PageRank sense for a subnetwork $\mathcal{G}_{\text{sub}}$ if the following constraints hold:*

$$((1-m)\mathbf{A}' + m\mathbf{J}')\,\mathbf{p}'^{\mathcal{G}'}(\mathcal{G}') = \mathbf{p}'^{\mathcal{G}'}(\mathcal{G}') \tag{58a}$$

$$\mathbf{J}'_{cm} = \mathbf{J}_{cm} \tag{58b}$$

$$\mathbf{J}'_{cc} = \mathbf{J}_{cc} \tag{58c}$$

$$\mathbf{A}'_{mm} = \mathbf{A}_{mm} \tag{58d}$$

$$\mathbf{A}'_{mc} = \mathbf{A}_{mc} \tag{58e}$$

$$\mathbf{A}'_{cm} = \mathbf{A}_{cm} \tag{58f}$$

$$\mathbf{A}'_{cc} = \mathbf{A}_{cc} \tag{58g}$$

$$\mathbf{A}'_{ce} = \mathbf{0} \tag{58h}$$

$$\mathbf{A}'_{ec} = \mathbf{0} \tag{58i}$$

where $\mathbf{p}'^{\mathcal{G}'}(\mathcal{G}')$ is the modified PageRank of $\mathcal{G}'$.

Constraints in (58) preserve the structure of $\mathcal{G}_{\text{sub}}$ and impose that all core and mid nodes get the same PageRank value over the equivalent network, i.e., $\mathbf{p}^{\mathcal{G}}(\mathcal{G}_{\text{sub}}) = \mathbf{p}^{\mathcal{G}'}(\mathcal{G}_{\text{sub}})$.

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Finally, the heuristic described before can be applied over the (simpler) equivalent network $\mathcal{G}'$—obtained following Theorem 8—in order to compute an approximation $v'(\mathcal{C})$ of (56).

The demonstration of Theorem 8 is omitted here. The reader is referred to [75] for a detailed discussion of the topic.

9.2.5 A coalitional control scheme based on the PageRank

Consider a dynamical network determined by the interaction of a set of subsystems $i \in \mathcal{N}$, whose behaviour can be modeled as the following discrete-time linear dynamics:

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) + d_i(k), \\ d_i(k) &= \sum_{j\in\mathcal{N}_i} A_{ij}x_j(k) + \sum_{j\in\mathcal{N}_i} B_{ij}u_j(k), \end{aligned} \tag{59}$$

where $x_i \in \mathbb{R}^{q_i}$ and $u_i \in \mathbb{R}^{r_i}$ with $i = 1, \dots, n$ are the local states and inputs, and $\mathcal{N}_i = \{j \in \mathcal{N} : A_{ij} \neq 0 \vee B_{ij} \neq 0\}$ is the set of coupled neighbors of node i. The variable d_i is the influence of the neighbors' states and inputs in the update of x_i. We also assume constraints on the state and the input as follows

$$x_i(k) \in \mathcal{X}_i, u_i(k) \in \mathcal{U}_i. \tag{60}$$

Moreover, assume that the interaction between the subsystems can be schematized over the undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. More specifically, the authors of [76] define the *aid graph* $\mathcal{G}_{\text{aid}} = (\mathcal{N}, \mathcal{E}_{\text{aid}})$, where $\mathcal{N}$ is the set of local controllers and $\mathcal{E}_{\text{aid}}$ is the set of edges representing the directed *aid links* that can be established among the nodes. We say that node i requests aid from node j if $(i,j) \in \mathcal{E}_{\text{aid}}$. Likewise, an *aid offer* from node i to node j is translated into a link $(i,j) \in \mathcal{E}_{\text{aid}}$. Notice that in both cases the goal is to increase the PageRank value of agent j. The goal of coalitional control is to produce a switching signal for the dynamical regrouping of local controllers. To this end, the PageRank and the *LinkRank* values are employed to calculate the relevance of the nodes and links inside the aid-network.

The key idea of the algorithm proposed by [76] is that local controllers evaluate their performance according to a certain criterion. Possible criteria include, e.g., the distance with respect to the setpoints, the value of the cost function optimized, the measured disturbances.

Here follows a sketch of the algorithm. At time step k:

1. Each local controller i measures its state and evaluates its performance according to a given criterion.
   - In case the performance is low the agent sends an aid request to its neighbors $\mathcal{N}_i$. This means that *aid links* (i,j) are formed for each $j \in \mathcal{N}_i$.
   - In case the performance is good the agent offers support to its set of neighbors $\mathcal{N}_i$. Each node $j \in \mathcal{N}_i$ accepts the proposal if its performance is low. This means that *aid links* (i,j) are formed for each $j \in \mathcal{N}_i$.

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2. The PageRank values of the nodes of the *aid network* are calculated by means of a distributed algorithm [82].
3. An average consensus algorithm is executed so that controllers become aware of the average *LinkRank* value.
4. Each node informs its neighbours about its PageRank value so that *LinkRank* values can be calculated.
5. Links with a *LinkRank* value greater than a certain threshold—that in turn is a function of the current LinkRank mean value—are enabled for control purposes. Links are assumed to be bidirectional. The resulting communication topology imposes a partition on the set of local controllers $\mathcal{N}$ into a set of disjoint coalitions.
6. Inside each coalition, local controllers have full communication and implement a distributed control algorithm [83] to calculate the input sequence in a coordinated fashion. Notice that different coalitions do not communicate with each other.

Remark 9 *In [76] it is assumed that distributed algorithms for PageRank and consensus can be used and computed within the sampling time. Nevertheless, the authors state that such assumption can be relaxed, since* LinkRank *values may be computed over a larger sampling period. Furthermore, this computations are performed in parallel with those regarding the synthesis of control actions.*

For further details, the reader is referred to [75, 76].

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10 Coalitional stability

In Section 6, we presented a greedy algorithm for coalition formation between control agents. It consists in a bottom-up approach, where the structure of the global control law is adapted to the time-variant coupling conditions of the system through the formation of coalitions *as outcome of a pairwise bargaining procedure*. Here comes the third and final step described in Section 4, i.e., the distribution of the value of a given coalition among its components. We present here a method guaranteeing coalition-wise stability, provided that the *core* of the associated transferable-utility (TU) game is *nonempty*.[16]

10.1 Introduction

We describe here a transfer scheme applicable to the autonomous coalition formation algorithm presented in Section 6 (see also [45]).[17] The discussion in Sections 5 and 6 referred to a scenario where the players aimed at minimizing their incurred control costs. Alternatively, in the remainder—without loss of generality—we consider rational players aiming at maximizing their individual payoff. Thus, player $i \in \mathcal{N}$ will choose an allocation p (associated to a given coalition $\mathcal{C}$) over p' (associated to a different coalition $\mathcal{C}'$) if $p_i > p'_i$. In case $p_i = p'_i$, we assume that the player will join the larger coalition.

The transfer scheme is performed among the control agents in a given coalition $\mathcal{C}$ in order to reach an agreement on the allocation of the joint benefit resulting from their cooperation. Such bargaining arise from the fact that some (subset of) agents $\mathcal{S} \subset \mathcal{C} \subseteq \mathcal{N}$ may claim a better individual payoff by not cooperating. Then, for the coalition $\mathcal{C}$ to be possible, the joint benefit has to be redistributed in order to satisfy such claims. We consider that all agents external to the claiming subset, i.e., all $i \in \mathcal{C} \setminus \mathcal{S}$, must support the demanded amount. After such amount is transferred and the claim is satisfied, a new demand may arise by a different subset $\mathcal{S}' \subset \mathcal{C}$ of agents, giving rise to an iterative process, that may be finite or not. An interesting property is that, under some assumption, such simple process converges to the core of the considered TU game.

10.2 TU transfer scheme

We begin by defining a *normalized* TU game, described by a set of players $\mathcal{N} = \{1, \ldots, N\}$ and a *characteristic function*

$$v : 2^{\mathcal{S}} \to \mathbb{R},$$

mapping each coalition $\mathcal{C} \subseteq \mathcal{N}$ to a normalized value satisfying

$$v(\varnothing) = 0, \quad v(\mathcal{N}) = 1, \quad v(\{i\}) = 0, \forall i \in \mathcal{N}. \tag{61}$$

[16] The reader is referred to [1] for a brief survey on the notions of game theory mentioned in the remainder.
[17] A rigorous formulation of such transfer scheme, along with the convergence demonstration, can be found in [84].

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In order to split the value of a coalition among its members, we define the set of *imputations* as

$$\mathcal{I} \triangleq \left\{ p = (p_1, \ldots, p_n) \in \mathbb{R}^n | \sum_{i \in \mathcal{N}} p_i = 1, p_i \geq 0, \forall i \in \mathcal{N} \right\}. \tag{62}$$

In words, (62) designates any vector payoff satisfying the *efficiency* and *individual rationality* properties.[18] Thus, an imputation guarantees to each member at least as much as it would achieve by playing as a singleton coalition.

For any given imputation p over $\mathcal{N}$, the *excess* is defined for each coalition $\mathcal{C} \subseteq \mathcal{N}$ as

$$e(\mathcal{C}, p) = v(\mathcal{C}) - \sum_{i \in \mathcal{C}} p_i, \tag{63}$$

with $e(\varnothing, p) = 0$. In words, the excess represents the difference between the value the members of $\mathcal{C}$ get by playing as the coalition $\mathcal{C}$ and the aggregate payoff they achieve by participating in a different agreement yielding the imputation p (e.g., by merging to another coalition $\mathcal{C}' \subseteq \mathcal{N} \setminus \mathcal{C}$, or by splitting into smaller coalitions).

The last concept we recall is the *core* of a coalitional TU game:

$$\mathcal{O} = \{ p \in \mathcal{I} | e(\mathcal{C}, p) \leq 0, \forall \mathcal{C} \subseteq \mathcal{N} \}, \tag{64}$$

representing the set of imputations that cannot be improved by any coalition $\mathcal{C} \subset \mathcal{N}$, i.e., any payoff p such that $\sum_{i \in \mathcal{C}} p_i \geq v(\mathcal{C})$. For the purpose of the transfer scheme that will be presented next, we restate (64) in terms of the *demand* against a generic allocation p by a the set of players $\mathcal{C}$.

Definition 10 (Demand) *A demand against* $p = (p_1, \ldots, p_n)$, $\sum_{i \in \mathcal{N}} p_i = 1$, *is a pair* $(\mathcal{C}, d)$ *where* $\varnothing \neq \mathcal{C} \subset \mathcal{N}$, *and* $d \triangleq (d_i)_{i \in \mathcal{C}}$ *is a vector satisfying*

$$d_i \geq 0 \tag{65a}$$

$$\sum_{i \in \mathcal{C}} d_i = e(\mathcal{C}, p). \tag{65b}$$

A demand is *essential* if $\sum_{i \in \mathcal{C}} d_i > 0$. Note that there cannot be an essential demand for the grand coalition $\mathcal{N}$.

In order to achieve the convergence of the proposed transfer scheme (see also [84]), we consider, for a given demanding set of agents $\mathcal{C}$, *uniform subdivision* of the demanded amount among the agents, i.e.,

$$d_i = \frac{e(\mathcal{C}, p)}{|\mathcal{C}|}. \tag{66}$$

A *satisfaction* to a demand $(\mathcal{C}, d)$ against p is an allocation s such that the agents in $\mathcal{C} \subset \mathcal{N}$ get as much as they are able to by playing as a standalone coalition $\mathcal{C}$. The satisfaction of such demand requires the transfer of an equivalent aggregate amount, equally drawn from the rest of agents. More formally, for every agent $i \in \mathcal{N}$,

$$s_i = \begin{cases} p_i + \frac{e(\mathcal{C}, p)}{|\mathcal{C}|}, & \text{if } i \in \mathcal{C}, \\ p_i - \frac{e(\mathcal{C}, p)}{|\mathcal{N} \setminus \mathcal{C}|}, & \text{if } i \in \mathcal{N} \setminus \mathcal{C}. \end{cases} \tag{67}$$

[18] The reader is referred to [1], Section 3, for further details.

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A transfer scheme is a sequence of allocation proposals $p^{(k)}$, $k \in \mathbb{N}$, such that $p^{(k+1)}$ is a satisfaction to a demand against $p^{(k)}$. Moreover, if there exists a $\hat{k}$ such that for all $k \geq \hat{k}$ we have $p^{(k)} = p^{(\hat{k})}$, the transfer scheme is *finite*, i.e., $e(\mathcal{C}^k, p^k) = 0$ for all $k \geq \hat{k}$.

Remark 11 *Notice that allocations produced in any intermediate iteration of the transfer scheme may not satisfy individual rationality. We assume that this is not an issue, as this constitutes as well a base for a new demand. Furthermore, if the game is convex, the overall dissatisfaction is lowered at each iteration and the transfer scheme converges to an imputation in the core in a finite number of steps.*[19]

[19] A game is defined to be convex if, given a player $i \in \mathcal{N}$ and any two coalitions $\mathcal{C}_a \subset \mathcal{C}_b \subseteq \mathcal{N} \setminus \{i\}$, it holds that $v(coal_a \cup \{i\}) - v(\mathcal{C}_a) < v(coal_b \cup \{i\}) - v(\mathcal{C}_b)$. In words, a game is convex when the marginal value derived by joining a coalition increases with the size of the coalition (see [1]).

11 System stability

This section presents an analysis of the stability properties of the coalitional control algorithms described in Sections 5 and 6. The analysis is based on the stability results available in the literature regarding switched systems, together with the input-to-state (ISS) stability of decentralized systems.

11.1 Introduction

A switched system is a system described by a finite collection of models, here referred to as *configurations* of the system.[20] The switching between different configurations is governed by given criteria (see, e.g., [85]). Let $\mathcal{I} \triangleq \{1, \dots, n_c\}$ be the set indexing each possible configuration of the system. For any configuration $i \in \mathcal{I}$, the behavior of the global state $x \in \mathbb{R}^n$ of a switched linear system can be described by the discrete-time model

$$x(k+1) = A^{(i)}x(k) + B^{(i)}u(k),\ i \in \mathcal{I}, \tag{68}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^q$ are the global state and input vectors, respectively; $A^{(i)} \in \mathcal{A}$ and $B^{(i)} \in \mathcal{B}$ are the relative transition matrices, belonging their respective sets of possible realizations $\mathcal{A}$ and $\mathcal{B}$.

The criteria upon which the switching is performed can be described through a piecewise constant map, $\sigma : \mathbb{N} \times \mathbb{R}^n \to \mathcal{I}$. So, at each time step k, there will be an active configuration $i = \sigma(k, x(k))$. [21]

In the remainder, we will consider the availability of a set of global linear feedback laws, each one associated to a given configuration of the system. Furthermore, we will assume unconstrained state and inputs.

11.2 Coalitional control as a switched multi-controller system

Consider the switched discrete-time linear system

$$x(k+1) = (A + BK^{(i)})x(k),\ i \in \mathcal{I}, \tag{69}$$

where x, A, and B are defined as in (68). Only one realization of the state and input matrices is considered, so that $\mathcal{A} \equiv A$ and $\mathcal{A} \equiv B$; $K^{(i)}(A, B) \in \mathcal{K}$ is a global feedback law stabilizing the system, and $\mathcal{K} : A \times B \to \mathbb{R}^{q \times n}$ denotes the family of linear feedback matrices stabilizing system (69). Each feedback law $K^{(i)}$ is associated to a given coalition structure $\mathcal{P} = \{C_1, \dots, C_{N_c}\}$, partitioning the system state vector into N_c disjoint subsets

$$x^{(i)} = (\xi_1^\intercal, \xi_2^\intercal, \dots, \xi_{N_c}^\intercal)^\intercal. \tag{70}$$

Consider the presence of a set $\mathcal{N} = \{1, \dots, N\}$ of agents, all in charge of the control of the system. In particular, for any $i \in \mathcal{I}$, each portion of the state vector can only be measured by a given coalition (singleton coalitions are included as well). Then, $K^{(i)}$ is obtained as the result of the following problem.

[20] We will see later that each configuration correspond to a given possible coalition structure of the control agents.

[21] Notice that, in general, the active configuration may depend as well on the previous configuration [85]. This occurs indeed in the two coalitional control schemes presented in Sections 5 and 6.

Problem 12 (Coalitional feedback) *Consider the discrete-time switched linear system described by* (69)*. According to the generic partition of the state vector given by* (70)*, the feedback matrix can be decomposed as*[22]

$$K^{(i)} = \begin{bmatrix} K_{11}^{(i)} & K_{12}^{(i)} & \cdots & K_{1p}^{(i)} \\ K_{21}^{(i)} & K_{22}^{(i)} & \cdots & K_{2p}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ K_{p1}^{(i)}, & K_{p2}^{(i)}, & \cdots, & K_{pp}^{(i)} \end{bmatrix} \tag{71}$$

Now, given the partial state feedback dictated by (70)*, the problem consists in finding a matrix* $K^{(i)}$ *such that any* $K_{ab}^{(i)} = 0$ *if* $a \neq b$ *(block-diagonal* $K^{(i)}$*), and the system* (69) *is stable.*[23]

Notice that problem 12 may not have a finite number of solutions.

Problem 13 *Consider all the conditions in problem 12. Define the control performance index associated to the control law* $K^{(i)}$ *as*

$$J^{(i)} = \sum_{k=0}^{\infty} x(k)^\intercal \left(Q + (K^{(i)})^\intercal R K^{(i)} \right) x(k), \tag{72}$$

Then, for a given configuration i*, find the matrix* $K^{(i)}$ *minimizing the difference* $(J^{(i)} - J^*)$*, where* J^* *is the optimal control performance associated to the centralized LQR feedback law (i.e., the one obtained with full state feedback).*

At this point, it is important to remark that it is not sufficient to independently guarantee the stability of each one of the configurations. Indeed, even if all the configurations are stable, there might exist some switching signals leading the system to evolve over divergent trajectories [85]. On the other hand, an interesting capability provided by switching is that a proper switching sequence between different unstable configurations can drive the system to a stable behaviour; however, this case is not discussed here.

It is clear that the way the system switches between different configurations, i.e., how agents join or leave coalitions, plays a fundamental role in the system behaviour. Attention has to be paid to the time instants at which agents are allowed to form, leave, or move over coalitions, and to which coalitions—determining a given partition of the global state feedback—are produced at any given time. In the remainder, we will try to delineate the necessary conditions that the evolving coalition structure must fulfill in order to ensure the overall system stability.

11.3 Stability of switched systems

A first approach to the problem is that of seeking for a Lyapunov function that holds for all configuration [85]. In particular, since we are dealing with linear models, we consider a quadratic Lyapunov function $V(x) = x^\intercal P x$, whose existence conditions can be posed in form of linear matrix inequalities

[22] The input matrix B is assumed here to be diagonal (or block-diagonal, with no coupling across partitioned subsets).

[23] Notice that $K^{(i)}$ will correspond to the decentralized (i.e., only local) feedback just for one configuration $i_0 \in \mathcal{I}$.

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(LMI). Therefore, we look for a matrix $P \in \mathbb{R}^{n \times n}$ such that the following constraints are satisfied for all $i \in \mathcal{I}$

$$P = P^{\intercal} > 0, \tag{73a}$$

$$(A + BK^{(i)})^{\intercal} P (A + BK^{(i)}) - P \leq 0, \tag{73b}$$

where (73a) requires matrix P to be symmetric and positive definite, and (73b) is a monotone decreasing condition on the value of the Lyapunov function at each pair of subsequent time steps. Efficient algorithms for the computation of (73) are available. The (block-diagonal) global feedback matrix $K^{(i)} \triangleq \text{diag}(K_1, \ldots, K_{N_c})$ can be calculated as the solution of an LMI problem, as shown, e.g., in [19,22]. If a solution to such structured global problem exists, the product of invariant sets for each individual subsystem constitutes an invariant set for the overall system, defined by matrix P. A distributed computation of such invariant sets may be carried out through the method described in [86]. This sets can be used as terminal constraints in the corresponding MPC problem. It is worth pointing out that the existence of a Lyapunov function common to all configurations is only sufficient to guarantee stability, constituting a very demanding condition in practice. Less stringent stability requirements may be found in [87].

11.4 Localized verification of the global cost

Alternatively, following the procedure of [36], whenever an agent or a group of agents plans to deviate from a given default (stable) behavior, the new control sequence is communicated to all the agents affected. These in return communicate their predicted cost variation, that the new input sequence produces. If the change is a decrease of the global cost, the proposal is accepted and implemented. In this way, only the control actions that actually improve the behavior of the overall system with respect to a predefined stable behavior are implemented. However, such method of guaranteeing stability requires a forced coordination between agents that do not belong to the same coalition. This may imply that agents in a coalition must keep acting according to their default plan as long as their proposals of deviating from it are not accepted by the affected agents outside the coalition. Moreover, in order to implement such strategy, each agent has to transmit their plans to all neighbors: this means that the system works on the basis of an information broadcast, which breaks the original assumption of autonomous negotiation between pairs of agents. Nevertheless, given that there is no negotiation, this is still less demanding—in terms of communications—than distributed algorithms based on information exchange (such as, e.g., [6]).

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12 Conclusion

This document presents two coalitional control architectures specifically developed to address the management of systems of systems. Furthermore, the population-based coalitional scenario developed within Task 1.3 of WP1 is introduced, as well as the price mechanisms from WP2 for coalition formation.[24] One of the critical points that have to be kept into consideration is the *diversity* characterizing the systems in object, yielding very complex interactions between the agents involved (see, e.g., the AYESA and HEP case studies contemplated within the DYMASOS project). In such settings, information about the relevance is of critical importance [20, 21]. Both cooperative and noncooperative game theory frameworks provide a vast knowledge base for this task [22, 23]. However, a critical computational complexity is naturally associated with the analysis and the control of evolving coalition structures emerging by the interaction of a significant (for real-world applications) number of agents [51]. Over Tasks 3.3 and 3.4, implementation aspects of coalitional algorithms, as well as information-related issues, have been addressed. Indeed, from the coalitional control point of view, it is critical to characterize the improvement provided by a broader knowledge of the system, and promote the formation of coalitions accordingly. Of course, this comes at a cost of a more intense information exchange. Again, the game theory appears as a natural provider of tools for the analysis of such problems, but the question of whether they can be adapted to large-scale real-time optimization still lingers on, due the intrinsic computational complexity associated with such analysis.

In the last part of this document, the reader can find a discussion of the stability of coalitional control settings from two different point of views: *(i)* analysis of the stability of coalitions, and the associated conditions of benefit transfers among cooperating controllers, and *(ii)* sufficient conditions for the stability of the system in control theoretic sense, i.e., guarantees of reaching the desired operation setpoint.

[24] Work carried out by the Automatic Control Laboratory team at ETH Zurich, and by the Faculty of Electrical Engineering and Computing (FER) team at University of Zagreb, respectively.

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